BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 77, Number 1, January 1971

DEFORMATIONS OF LIE SUBGROUPS

BY R. W. RICHARDSON, JR.[1]

Communicated by Louis Auslander, April 3, 1970

1. **Introduction.** This note is an announcement of results concerning the local deformation theory of subgroups of a Lie group. Let G be a real (resp. complex) Lie group and let M be a real (resp. complex)-analytic manifold. Roughly speaking, an analytic family of Lie subgroups of G, parametrized by M, is an analytic submanifold $\mathcal{H}$ of $G\times M$ such that each "fibre" H_t $(t\in M)$ is a Lie subgroup of G; here the "fibre" H_t is defined by $\mathcal{H}\cap(G\times\{t\})=H_t\times\{t\}$. (See §2 for a precise definition of an analytic family of Lie subgroups.) Our main result concerning such families is

THEOREM A. *Let $\mathcal{H}=(H_t)_{t\in M}$ be an analytic family of Lie subgroups of G, let $t_0\in M$ and let $H=H_{t_0}$. Let K be a Lie subgroup of H such that the component group K/K^0 is finitely generated and such that the Lie group cohomology space $H^1(K, \mathfrak{g}/\mathfrak{h})$ vanishes. Then there exists an open neighborhood U of t_0 in M and an analytic map $\beta: U\to G$ such that $K\subset\beta(t)H_t\beta(t)^{-1}$ for every $t\in U$.*

Here $\mathfrak{g}$ (resp. $\mathfrak{h}$) denotes the Lie algebra of G (resp. H) and the K-module structure of $\mathfrak{g}/\mathfrak{h}$ is determined by the adjoint representation of K on $\mathfrak{g}$.

Theorem A generalizes the result of A. Weil [6, p. 152] which states that if Γ is a discrete, finitely generated subgroup of G such that $H^1(\Gamma, \mathfrak{g})=0$, then Γ is "rigid". It also generalizes results of the author [4], [5] on deformations of subalgebras of Lie algebras to the case of Lie subgroups. The proof of Theorem A depends heavily on the analyticity assumptions, although we suspect that the C^∞ version of the theorem is also true.

If G acts as an analytic transformation group on the analytic manifold M and if all orbits of G on M have the same dimension, then it can be shown that the connected isotropy groups $(G_t^0)_{t\in M}$ form an analytic family of Lie subgroups of G, and hence Theorem A applies. For example, let K be a maximal compact subgroup of G_t^0. Then

AMS 1969 *subject classifications.* Primary 2250; Secondary 2240, 1450.
Key words and phrases. Lie groups, deformations, analytic transformation groups, algebraic transformation groups.
[1] Partial support received from National Science Foundation Grant GP-11844.