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AN ALGEBRA OF GENERALIZED FUNCTIONS ON AN OPEN INTERVAL; TWO-SIDED OPERATIONAL CALCULUS

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Let Ω be an open subinterval of the real line; suppose that $0 \in \Omega$. The purpose of this announcement is to describe an injection of $L^{\mathrm{loc}}(\Omega)$ into a commutative algebra of operators. The injection is a useful substitute for the two-sided Laplace transformation; in case Ω is the whole real line, the injection can be extended to a space $\mathfrak{B}$ of distributions whose supports may be all of $(-\infty, \infty)$ (there are no growth restrictions: see §7). If the distributional derivative of an arbitrary distribution R belongs to the space $\mathfrak{B}$, then R also belongs to $\mathfrak{B}$ and R has an initial value (because R equals a continuous function in some left-neighborhood of the origin). Thus, it is possible to assign initial conditions to the unknown distribution in a differential equation whose right-hand side belongs to the space $\mathfrak{B}$: see 7.3.

1. **Preliminaries.** If $f_1(\ )$ and $f_2(\ )$ belong to the space $L^{\mathrm{loc}}(\Omega)$ (of all the complex-valued functions which are integrable on each compact subinterval of the open set Ω), we denote by $f_1 \wedge f_2(\ )$ the function defined by

$$(1.1) \qquad f_1 \wedge f_2(t) = -\int_t^0 f_1(t-u) f_2(u)\,du \qquad \text{(for all } t \text{ in } \Omega).$$

2. **The space of test-functions.** Let $W\Omega$ be the space of all the complex-valued functions which are infinitely differentiable on Ω and whose every derivative vanishes at the origin. Thus, $w(\ )$ belongs to $W\Omega$ if $w(\ ) \in C^\infty(\Omega)$ and $w^{(k)}(0) = 0$ for every integer $k \geqq 0$.

2.1. *The space of generalized functions.* Let $\mathfrak{A}\Omega$ be the space of all the linear operators A which map $W\Omega$ into $W\Omega$ such that the equation

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