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BOOK REVIEWS

Convergence of probability measures by P. Billingsley. Translated by R. M. Dudley. Wiley Series in Probability and Mathematical Statistics, John Wiley, New York, 1968. xii+253 pp. $12.50.

The author's preface gives an outline: "This book is about weak-convergence methods in metric spaces, with applications sufficient to show their power and utility. The Introduction motivates the definitions and indicates how the theory will yield solutions to problems arising outside it. Chapter 1 sets out the basic general theorems, which are then specialized in Chapter 2 to the space $C[0, 1]$ of continuous functions on the unit interval and in Chapter 3 to the space $D[0, 1]$ of functions with discontinuities of the first kind. The results of the first three chapters are used in Chapter 4 to derive a variety of limit theorems for dependent sequences of random variables."

The book develops and expands on Donsker's 1951 and 1952 papers on the invariance principle and empirical distributions. The basic random variables remain real-valued although, of course, measures on $C[0, 1]$ and $D[0, 1]$ are vitally used. Within this framework, there are various possibilities for a different and apparently better treatment of the material.

More of the general theory of weak convergence of probabilities on separable metric spaces would be useful. Metrizability of the convergence is not brought up until late in the Appendix. The close relation of the Prokhorov metric and a metric for convergence in probability is (hence) not mentioned (see V. Strassen, Ann. Math. Statist. 36 (1965), 423–439; the reviewer, ibid. **39** (1968), 1563–1572). This relation would illuminate and organize such results as Theorems 4.1, 4.2 and 4.4 which give isolated, *ad hoc* connections between weak convergence of measures and nearness in probability.

In the middle of p. 16, it should be noted that $C^*(S)$ consists of signed measures which need only be finitely additive if S is not compact. On p. 239, where the author twice speaks of separable subsets having nonmeasurable cardinal, he means "discrete" rather than "separable."

Theorem 1.4 is Ulam's theorem that a Borel probability on a complete separable metric space is tight. Theorem 1 of Appendix 3 weakens completeness to topological completeness. After mentioning that probabilities on the rationals are tight, the author says it is an