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FINITE DIMENSIONAL *H*-SPACES[1]

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1. **Introduction.** An *H-space* is a space X with base point e equipped with a continuous function

$$X \times X \xrightarrow{\mu} X$$

such that $\mu(x, e) = x = \mu(e, x)$ for all $x \in X$. This condition can also be formulated as follows. The *wedge* $X \vee X$ of X is defined by

$$X \vee X = (X \times e) \cup (e \times X) \subset X \times X,$$

and the *folding map* ∇ is given by

$$\nabla(x, e) = x, \qquad \nabla(e, x) = x, \qquad \nabla : X \vee X \to X.$$

The H-space condition is then that the following diagram commutes

$$\begin{tikzcd} X \times X \arrow[r, "\mu"] & X \\ X \vee X \arrow[u, "i"] \arrow[ur, "\nabla"'] & \end{tikzcd}$$

where i is the inclusion map. This formulation has the advantage that we can now relax the condition and just require that the diagram be homotopy commutative; i.e., that the maps $\mu \circ i$ and are homotopic, $\mu \circ i \sim \nabla$.

When one is considering different H-space structures μ, μ' on an H-space X this distinction between the diagram commuting and homotopy commutating is important. But we will be concerned solely

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[1] An address delivered to the American Mathematical Society at the University of Wisconsin on April 18, 1970, by invitation of the Committee to Select Hour Speakers for Western Sectional meetings.

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Key words and phrases. H-space, Lie group, mod p cohomology, mixed homotopy types, fiber bundles.

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[3] The author felt no need to expand the talk to a survey of H-spaces because of the monograph of Stasheff, "H-spaces from the homotopy point of view," Springer, 1970. The only exceptions are §8, which notes some recent results not covered in the talk (nor in Stasheff's monograph), and a bibliography much more extensive than just references made in the paper.