ON THE AVERAGE ORDER OF IDEAL FUNCTIONS AND OTHER ARITHMETICAL FUNCTIONS

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ABSTRACT. We consider a large class of arithmetical functions generated by Dirichlet series satisfying a functional equation with gamma factors. We state a general O-theorem for the average order of these arithmetical functions and apply the result to ideal functions of algebraic number fields.

Landau [4] and Chandrasekharan and Narasimhan [3] have proved O-theorems for the average order of a large class of arithmetical functions. The method of proof uses finite differences and is due to Landau. Often, it is desired to have an O-theorem where the error term is a function of a certain parameter, which is the discriminant, for example, in the case of an algebraic number field. We state here a general O-theorem of this type. The method of proof is a slight modification of Landau's mentioned above.

We briefly indicate the arithmetical functions under consideration. For a more complete description see [3].

Let $\{a(n)\}$ and $\{b(n)\}$ be two sequences of complex numbers not identically zero. Let $\{\lambda_n\}$ and $\{\mu_n\}$ be two strictly increasing sequences of positive numbers tending to ∞. Put $s=\sigma+it$ with σ and t both real. We assume that

$$\varphi(s) = \sum_{n=1}^{\infty} a(n)\lambda_n^{-s} \quad \text{and} \quad \psi(s) = \sum_{n=1}^{\infty} b(n)\mu_n^{-s};$$

each converge in some half-plane and satisfy the functional equation

$$\Delta(s)\varphi(s) = \Delta(r-s)\psi(r-s),$$

where r is real and

$$\Delta(s) = \prod_{\nu=1}^{N} \Gamma(\alpha_\nu s + \beta_\nu),$$

where $\alpha_\nu > 0$ and β_ν is complex, $\nu = 1, \cdots, N$.

In the sequel A always denotes a positive number not necessarily the same with each occurrence.

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