INTRINSICALLY ERGODIC SYSTEMS

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Let ϕ be a continuous mapping of a compact metric space X into itself and M_ϕ the set of normalized ($\mu(x)=1$) ϕ-invariant measures on X. Kryloff and Bogoliouboff introduced the notion of *unique ergodicity* to describe the situation in which M_ϕ reduces to a single point. We shall present here a generalization of this concept, illustrate its usefulness and discuss some examples.

1. Denote the measure entropy of ϕ with respect to $\mu \in M_\phi$ by $h_\mu(\phi)$ and set

$$\bar{h}(\phi) = \sup_{\mu \in M_\phi} h_\mu(\phi).$$

DEFINITION. If $\bar{h}(\phi) < +\infty$ and there exists a unique $\bar{\mu} \in M_\phi$ such that

$$h_{\bar{\mu}}(\phi) = \bar{h}(\phi)$$

then (X, ϕ) is said to be an *intrinsically ergodic system* (i.e.s.).

Clearly a uniquely ergodic system with finite entropy is an i.e.s.; that the converse is not true may be seen already from the example of the bilateral 2-shift which is obviously not uniquely ergodic but is an i.e.s. We shall sketch two proofs of this well-known fact, since the methods are useful for later generalizations.

1. (After Parry [9].) Let α denote the basic partition of $\times_{-\infty}^{\infty}\{0, 1\}$ into two sets, A_0, A_1 where A_i consists of all sequences with an i in the zeroth place. α is a generator for the shift with respect to any regular shift invariant measure m (see [10] for the facts and notations of entropy theory used here) and one deduces from the fact that equality holds in

$$H_m(\alpha \vee \phi^{-1}\alpha) \leqq H_m(\alpha) + H_m(\phi^{-1}\alpha)$$

only if α and $\phi^{-1}\alpha$ are independent, that a maximizing m must be such that the coordinate functions are independent. Then an elementary computation reveals that there is a unique measure maximizing the entropy.

2. (After [2].) One observes that the number of sets in $\alpha, \phi^{-1}\alpha, \cdots, \phi^{-n+1}\alpha$ is 2^n and hence

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