A THEOREM ON COVARIANT CONSTANT (1, 1)-TENSOR FIELDS

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This note announces the proof of the following

THEOREM. *Suppose M is an n-dimensional C^∞ manifold with a connection ∇ and a $(1, 1)$-tensor field J which satisfy $\nabla J=0$. Then M admits a Riemann structure G and a second connection $\tilde{\nabla}$ satisfying $\tilde{\nabla}G=0$ and $\tilde{\nabla}J=0$.*

We have a proof which explicitly constructs such a G and $\tilde{\nabla}$ in terms of distributions on M defined by the semisimple and nilpotent parts of J. An alternative proof using the theory of principle fibre bundles [**1**] stems from showing that for any endomorphism $J: \mathbf{R}^n \rightarrow \mathbf{R}^n$, the centralizer of J in $\mathrm{GL}(n)$ is diffeomorphic to the manifold product of a space of orthogonal commutants of J (with respect to a certain inner product) and a Euclidean space. The decomposition was initially derived using matrix methods, but Dr. S. Halperin has suggested a shorter proof of the decomposition using a theorem of K. Iwasawa [**2**].

Implicit in the theorem is that the adjoint J^* of J with respect to G satisfies $\tilde{\nabla}J^*=0$, or equivalently, that J^* commutes with the specified orthogonal commutants of J. In the special case that J is an almost tangent structure ($J^2=0$, $2 \cdot \operatorname{rank} J=n$), it turns out that $\mathcal{J}_1=J+J^*$ and $\mathcal{J}_2=J-J^*$ satisfy $\mathcal{J}_1^2=-\mathcal{J}_2^2=I$ and $\mathcal{J}_1\mathcal{J}_2=-\mathcal{J}_2\mathcal{J}_1$. By first choosing a Riemann structure, C. S. Houh [**3**] was able to derive the relations for $\mathcal{J}_1$ and $\mathcal{J}_2$ and thereby the $\tilde{\nabla}$ of the theorem. Also studying almost tangent structures, H. Wakakuwa and S. Hashimoto [**4**] were able to specify an inner product on $\mathbf{R}^n$ and derive using matrices the above decomposition of the centralizer of J in $\mathrm{GL}(n)$. They further noted the commutativity of J^* (and hence $\mathcal{J}_1$ and $\mathcal{J}_2$) with the orthogonal commutants of J.

Another application of the theorem shows that if M admits a cyclic J with real eigenvalues, then M admits a flat connection. D. E. Blair and A. P. Stone [**5**] observed that this fact with the stronger assumptions that ∇ is a Riemann connection and that J has n distinct real eigenvalues.

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