INVERSE LIMITS AND MULTICOHERENCE

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1. Introduction. In this paper we state some results concerning inverse limits and multicoherence and give applications to hyperspaces and inverse limits of special types of spaces. The proofs of these and other related results will appear elsewhere.

By a *metric continuum* we mean a nonempty compact connected metric space. A space X is said to have *property* (b) (see [2, p. 63] or [10, p. 226]) if and only if, given a continuous function f from X into the unit circle in the plane, there exists a continuous real valued function α defined on X such that $f(x)=e^{i\alpha(x)}$ for each $x\in X$. If X is a metric continuum, then we say that X has *multicoherence degree k* (see [3] or [10, p. 83]) provided l.u.b. $\{r(X_1, X_2): X_1$ and X_2 are subcontinua of X with $X_1\cup X_2=X\}=k$, where $r(X_1, X_2)$ denotes one less than the number of components of $X_1\cap X_2$. The multicoherence degree of X is denoted by $r(X)$. We note that $r(X)=0$ is equivalent to X being unicoherent. It is well known that if X is a metric continuum with property (b), then X is unicoherent (see [2, p. 69] or [10, p. 227]), but not conversely.

All inverse systems considered in this paper are countable and the inverse limit of an inverse sequence $\{X_i, f_i\}_{i=1}^{\infty}$ is denoted by **proj lim** $\{X_i, f_i\}_{i=1}^{\infty}$. For notation and terminology relating to inverse limits, see [1].

2. Basic theorems.

THEOREM 1. *If* $X=$ proj lim$\{X_i, f_i\}_{i=1}^{\infty}$ *and each space* X_i *is a metric continuum with property* (b), *then* X *has property* (b).

THEOREM 2. *If* $X=$proj lim$\{X_i, f_i\}_{i=1}^{\infty}$ *where, for each* $i=1, 2, \cdots$, X_i *is a metric continuum,* $r(X_i)\leqq k$, *and* f_i *is a mapping of* X_{i+1} *onto* X_i, *then* $r(X)\leqq k$.

We note that if each of the bonding maps f_i in Theorem 2 are monotone and $r(X_i)=k$ for all $i=1, 2, \cdots$, then $r(X)=k$. This observation leads to very simple proofs of Theorem 4.8 and Theorem 4.11 of [1].

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