CERTAIN TOPOLOGICAL GROUPS ARE TYPE I[1]

BY ROBERT R. KALLMAN

Communicated by Gian-Carlo Rota, September 22, 1969

The purpose of this paper is to prove a theorem which roughly states that if G is a topological group with a "sufficiently large" Type I subgroup H, then G itself is also Type I. The main tool in the proof of this theorem is a result of Tomiyama [3] which gives a sufficient condition for a subalgebra of a Type I von Neumann algebra to be Type I. As far as the author knows, this result of Tomiyama has not been applied previously to group representations.

The main result is the following theorem. We note that in the special case in which G and H are separable locally compact groups, H is a closed normal subgroup of G, and G/H is compact, the following theorem may be deduced from Mackey [1].

THEOREM 1. *Let G be a topological group and let H be a closed subgroup of G. Suppose that G/H is locally compact and possesses a nonzero, finite, regular, G-invariant Borel measure. Then G is Type* I *if H is Type* I.

Using methods first developed by Sakai [2], Tomiyama [3] proved the following lemma. It is of fundamental importance in what follows.

LEMMA 2. *Let $\mathbf{R}$ and $\mathbf{S}$ be von Neumann algebras with $\mathbf{S}\subseteq\mathbf{R}$. Suppose that there exists a linear mapping $P:\mathbf{R}\rightarrow\mathbf{S}$ with the following properties:*

(1) $P(I)=I$;

(2) $P(A)\geqq 0$ *if* $A\geqq 0$;

(3) $P(ABC)=AP(B)C$ *(A and C in $\mathbf{S}$, B in $\mathbf{R}$)*;

(4) *P is continuous in the ultraweak operator topology. Then $\mathbf{S}$ is Type* I *if $\mathbf{R}$ is Type* I.

In what follows, let $\pi:G\rightarrow G/H$ be the natural quotient mapping.

Let U be a strongly continuous unitary representation of G on the Hilbert space $\mathbf{K}$. Let $\mathbf{R}(G)$ (respectively, $\mathbf{R}(H)$) be the von Neumann algebra on $\mathbf{K}$ generated by $[U(a)\mid a \text{ in } G]$ (respectively, by $[U(a)\mid a \text{ in } H]$). We have that $\mathbf{R}(H)\subseteq\mathbf{R}(G)$. $\mathbf{R}(H)$, and therefore $\mathbf{R}(H)'$, is Type I by assumption.

AMS Subject Classifications. Primary 4665.

Key Words and Phrases. von Neumann algebras, operator theory.

[1] Research was supported in part by NSF Grant GP-9141.