ABELIAN QUOTIENTS OF THE MAPPING CLASS GROUP OF A 2-MANIFOLD

BY JOAN S. BIRMAN

Communicated by Raoul Bott, July 7, 1969

Let T_g be a closed, orientable 2-manifold of genus g, and let M_g be the mapping class group of T_g, that is the group of orientation-preserving homeomorphisms of $T_g \to T_g$ modulo those isotopic to the identity. The following theorem was proved by D. Mumford in [6]: If $[M_g, M_g]$ is the commutator subgroup of M_g, then $A_g = M_g/[M_g, M_g]$ is a finite cyclic group whose order is a divisor of 10. We give a very brief and elementary reproof of Mumford's theorem, and at the same time improve his result to show that the order of A_g is 2 if $g \geqq 3$.

Generators for M_g are well known, and a particularly convenient set is given by W. B. R. Lickorish in [3]. Lickorish's generators are "screw maps" about closed curves on the surface T_g (the definition of a screw map is the same as that in [6]), and Lickorish shows that the screw maps about the curves $\{u_i, z_i, c_j;\ 1 \leqq i \leqq g,\ 1 \leqq j \leqq g-1\}$ in Figure 1 generate M_g.

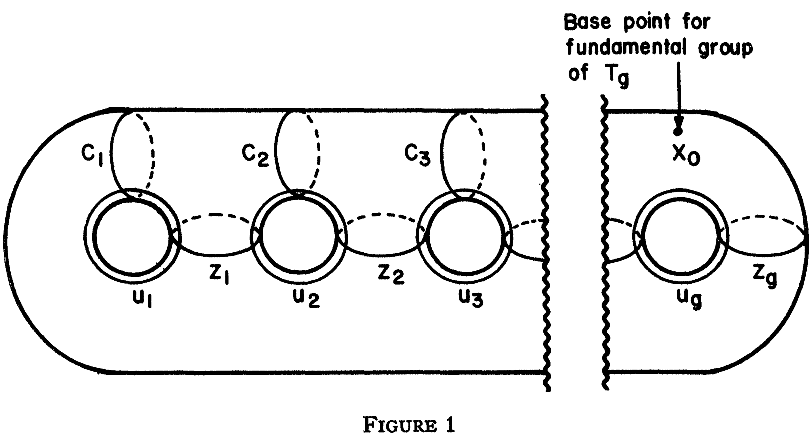

FIGURE 1

By a well-known result [5] the group M_g is isomorphic to a group of automorphism classes (cosets of the subgroup of inner automorphisms in the group of all automorphisms) of the fundamental group

AMS Subject Classifications. Primary 3045, 2022; Secondary 3080, 1451.

Key Words and Phrases. Mapping class group, homeotopy groups, automorphism group of the fundamental group of a 2-manifold, Teichmuller modular group, screw map, twist map.