CAUCHY PROBLEMS INVOLVING A SMALL PARAMETER

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The purpose of this note is to indicate how certain asymptotic methods developed for ordinary differential equations can be extended and applied to initial-boundary value problems for nonlinear parabolic and hyperbolic equations. This is done by considering the initial-boundary value problem as a Cauchy problem for an ordinary differential equation in an abstract space.

We consider the initial value problem

$$(1)\qquad \epsilon(dv/dt) - A(t, \epsilon)v = f(t, v, \epsilon), \qquad 0 \leqq t \leqq T, \quad v(0) = \mathring{v}(\epsilon)$$

where v is an element of a Banach space E and $\epsilon > 0$ is a small parameter. The (possibly unbounded) linear operators A are assumed to have a common domain of definition $\mathfrak{D}$ independent of (t, ϵ), and the function f is assumed to have continuous derivatives with respect to t, ϵ and continuous Fréchet derivatives with respect to v. Finally, $\mathring{v}(\epsilon) \in \mathfrak{D}$ has continuous derivatives with respect to ϵ.

We will outline here a method for finding an expansion for the solution of (1) which is valid as $\epsilon \to 0$.

1. **Formal method.** We begin by formally describing the procedure. These steps will be justified by Theorems 1–3. Suppose

(I) the operator $A(t, \epsilon)$ has a bounded inverse for each (t, ϵ) and $A(t, \epsilon)A^{-1}(0, 0)$ has continuous derivatives with respect to (t, ϵ).

Assuming for the moment that (1) has a solution for $\epsilon > 0$, we differentiate (1) successively with respect to ϵ and set $\epsilon = 0$ in the results. This gives the system of equations

$$(2a)\qquad - A(t, 0)v_0 = f(t, v_0, 0)$$

$$(2b)\qquad -[A(t, 0) + f_r(t, v_0(t), 0)]v_r = R_r(t), \qquad r = 1, 2, \cdots,$$

for the coefficients v_r of the Taylor expansion of v about $\epsilon = 0$. Next, we make the change of variables $t = \epsilon\tau$ in (1):

$$(3)\qquad dV/d\tau - A(\epsilon\tau, \epsilon)V = f(\epsilon\tau, V, \epsilon), \qquad V(0) = \mathring{v}(\epsilon).$$

By differentiating this successively with respect to ϵ and setting $\epsilon = 0$ in the results, we get

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