NOTES TOWARDS THE CONSTRUCTION OF NONLINEAR RELATIVISTIC QUANTUM FIELDS. II: THE BASIC NONLINEAR FUNCTIONS IN GENERAL SPACE-TIMES

BY IRVING SEGAL[1]

Communicated by Murray Protter, September 17, 1969

1. In earlier work [1], [2], the generalized nonlinear functions which enter formally into certain nonlinear relativistic quantized partial differential equations in a two-dimensional space-time were given mathematical formulation and treatment. The theory of these highly singular functions, and in particular their locality, when combined with hyperbolicity ideas implemented by the use of the Lie-Trotter formula, gave a means of adapting the treatment of quantum field dynamics in terms of a group of automorphisms of a C^*-algebra, developed initially in [3], to the equations in question. The present work describes an extension of the theory of these basic functions (or "renormalized products of quantum fields"), to general space-times and laws of dependence of the energy on the momentum, relativistic theory in a Minkowski space of any dimension being a rather special case. The general dynamical implications of these results will be treated later.

2. Since apart from the recent work [1], [2], [4], [5], [6] there is little mathematical literature on renormalized powers of quantum fields, it may be helpful to describe briefly the background of the subject. Renormalized products of quantum variables in general, and of quantum fields in particular, arose and were studied soon after the introduction of quantum mechanics. The ambiguities in the calculus of operators satisfying "canonical commutation relations" became more serious in the case of quantum fields, where they appeared in part as "infinities" or so-called "divergences." The usage in the physical literature was standardized as a result of work of Wick [6], whose mathematical core as now perceived is a theorem in finite-dimensional algebra (cf. Theorem 1.3 of [2]). Its extensive application in the physical literature has been to situations involving a continuum of variables (typically, the "values" of a generalized function on R^n); these are treated by formal analogy with the finite-dimensional, purely algebraic, situation.

AMS Subject Classification. Primary 2846, 8147; Secondary 3495, 4665.

Key Words and Phrases. Renormalized product, regularity domain, temporal smoothing, quantum field.

[1] Research sponsored in part by the Office of Scientific Research.