# SPECTRAL PROPERTIES OF HIGHER DERIVATIONS ON SYMMETRY CLASSES OF TENSORS

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Let $V$ be a finite dimensional vector space over a field $R$, and let $T\colon V \to V$ be a linear transformation on $V$. The transformation $T$ may be extended in a unique way to a derivation $\Omega_1(T)$ of the tensor algebra $\otimes V = \sum_{p=0}^{\infty} \otimes_1^p V$ by defining $\Omega_1(T)$ on each of the homogeneous components $\otimes_1^p V$ by

$$\Omega_1(T)v_1 \otimes \cdots \otimes v_p = \sum_{j=1}^{p} v_1 \otimes \cdots \otimes v_{j-1} \otimes Tv_j \otimes v_{j+1} \otimes \cdots \otimes v_p$$

when $p > 1$, and $\Omega_1(T) = 0$ if $p = 0$, $\Omega_1(T) = T$ if $p = 1$ (Greub [2, p. 67]).

In this announcement we outline an extension of this definition to higher order derivations in general symmetry classes of tensors. We also state some of the eigenvalue properties of higher derivations and show how these may be applied to some classical matrix problems. Let $H$ be a subgroup of $S_p$ and let $\chi$ be a character of degree 1 on $H$. (We assume that the order of $H$ exceeds the characteristic of $R$.) If $U$ is a vector space over $R$ and $\phi(v_1, \cdots, v_p)$ is a $p$-multilinear function on the Cartesian product $\times_1^p V$ to $U$, then $\phi$ is said to be symmetric with respect to $H$ and $\chi$ if

$$\phi(v_{\sigma(1)}, \cdots, v_{\sigma(p)}) = \chi(\sigma)\phi(v_1, \cdots, v_p)$$

for any $\sigma \in H$ and arbitrary vectors $v_i \in V$. A pair $(P, \mu)$ consisting of a vector space $P$ over $R$ and a $p$-multilinear function $\mu\colon \times_1^p V \to P$, symmetric with respect to $H$ and $\chi$, is a symmetry class of tensors associated with $H$ and $\chi$ if

(i) $\langle \operatorname{rng} \mu \rangle = P$ (i.e. the linear closure of the range of $\mu$ is $P$);

(ii) for any vector space $U$ over $R$ and any $p$-multilinear $\phi$ symmetric with respect to $H$ and $\chi$, there exists a linear function $h\colon P \to U$ such that $\phi = h\mu$.

$$\begin{tikzcd} \overset{p}{\underset{1}{\times}} V \arrow[rr, "\mu"] \arrow[dr, "\phi"'] & & P \arrow[dl, "h"] \\ & U & \end{tikzcd}$$

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