# THE NONEXISTENCE OF BRANCH POINTS IN THE CLASSICAL SOLUTION OF PLATEAU'S PROBLEM

BY ROBERT OSSERMAN[1]

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The result to be proved is the following.

THEOREM. *Let $C$ be an arbitrary Jordan curve in $R^3$. Denote by $\Delta$ the closed unit disk in $R^2$. Then there exists a regular minimal surface $S$ of the type of the disk spanning $C$. Specifically, there exists a map*

$$h\colon \Delta \to R^3$$

*satisfying*

(i) *$h$ is continuous in $\Delta$;*

(ii) *$h$ maps the boundary of $\Delta$ homeomorphically onto $C$;*

(iii) *each component $h_k$ of $h$ is a harmonic function in the interior of $\Delta$, hence the real part of an analytic function $\Phi_k$. The functions $\Phi_k$ satisfy*

$$\sum_{k=1}^{3} (\Phi_k')^2 \equiv 0 \tag{1}$$

*and*

$$\sum_{k=1}^{3} |\Phi_k'|^2 \neq 0 \quad \textit{everywhere.} \tag{2}$$

(iv) *the surface $S$ defined by $h$ has least area among all maps of $\Delta$ into $R^3$ satisfying* (i) *and* (ii); *if the area of $S$ is infinite, then every interior portion of $S$ has minimum area with respect to its own boundary curve.*

It is well known that condition (iii) implies that $h$ is a conformal immersion of the interior of $\Delta$ onto a regular minimal surface. (See for example [**1**, §II.18].)

The above theorem was proved by Douglas and Radó (see [**1**, §§VI.1–7]) *except* for condition (2). Since the functions $\Phi$ are analytic and not all constant by (ii), it follows that condition (2) can fail at most at isolated points. Such points are called *branch points*. It has remained an open question whether these branch points actually

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