MINIMAL VARIETIES[1]

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ABSTRACT. This is a survey article, reporting on recent results in the theory of minimal varieties in euclidean space, and including a number of theorems on minimal submanifolds of spheres.

Introduction. It was exactly 100 years ago, in 1868, that Beltrami presented the first general survey of the theory of minimal surfaces [4]. This survey has been referred to by Blaschke [5, p. 118] as representing the "stormy youth" of the subject, in contrast to its "tired old age" in the nineteen thirties. Although I would take issue with both of Blaschke's characterizations, I think it incontestable that the last ten years have seen a "stormy rebirth" of the theory of minimal surfaces. One of the most striking developments, although certainly not the only one, has been the creation of a theory of higher-dimensional minimal varieties. Since several recent surveys (Nitsche [35], Osserman [36(c)]) have been devoted to two-dimensional minimal surfaces, the goal here will be to concentrate on the higher-dimensional case, and restrict the discussion of two-dimensional surfaces to some of the most recent results. Furthermore, we shall discuss only minimal varieties in euclidean spaces, except for §6, which deals also with minimal submanifolds of spheres. Since much recent work has been devoted to minimal submanifolds of spheres, and some to arbitrary Riemannian manifolds, we have included these topics in the list of references. Note in particular the set of lecture notes by Chern [10(b)].

There are many points of view from which minimal varieties may be studied. The emphasis here will be on their differential-geometric properties, and on the associated differential equations. For detailed discussions relative to the calculus of variations and to measure the-

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