AN APPLICATION OF K-THEORY TO EQUIVARIANT MAPS

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1. **Introduction.** Let p be a prime and Z_p the cyclic group of order p. Denote by (S^{2n+1}, T_p) the free action of Z_p on S^{2n+1} given by

$$T_p(z_1, \cdots, z_{n+1}) = (\lambda z_1, \cdots, \lambda z_{n+1}) \quad \text{where} \quad \lambda = \exp(2\pi i/p).$$

Then given any pair (X, T) consisting of a finite complex X and a fixed point free transformation $T: X \to X$ of period p, one might ask for the least value of n for which there is an equivariant map of (X, T) into (S^{2n+1}, T_p). Questions of this type have been previously investigated [**3**], [**4**], [**5**] with particular emphasis on the case $p=2$.

It is the purpose of this note to describe a method for using K-theory to approach this problem for certain actions on lens spaces.

2. **Preliminaries.** Let BZ_{p^r} be a classifying space for the group Z_{p^r}, taken to be a CW complex whose odd dimensional skeleta are the lens spaces $L(p^r, 2n+1) = S^{2n+1}/T_{p^r}$ where T_{p^r} is defined analogously to T_p. Let BS^1 be a classifying space for the circle group whose even dimensional skeleta are complex projective spaces $CP(n)$.

Denote by K^* and K_* the Z_2-graded cohomology and homology theories arising from the unitary spectrum (see [**6**] for details).

(2.1) THEOREM. *There is a short exact sequence of groups*

$$0 \to K^0(BS^1) \xrightarrow{\alpha} K^0(BS^1) \xrightarrow{\gamma} K^0(BZ_{p^r}) \to 0$$

where $K^0(BS^1) \approx Z[[\mu]]$ is a power series ring in one variable, γ is a ring homomorphism, and α is given by multiplication by $[(\mu+1)^{p^r}-1]$.

(2.2) THEOREM. *There exists a short exact sequence of groups*

$$0 \to \tilde{K}_0(BS^1) \xrightarrow{\beta} K_0(BS^1) \to K_1(BZ_{p^r}) \to 0$$

where $K_0(BS^1)$ is the free abelian group generated by $\{g_i\}_{i=0}^{\infty}$ and

$$\beta(g_k) = g_{k-p^r} + \binom{p^r}{1} g_{k-p^r+1} + \cdots + \binom{p^r}{p^r-1} g_{k-1},$$

where $g_k = 0$ whenever $k<0$.