SURGERY IN $M\times N$ WITH $\pi_1 M\neq 1$

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1. We announce here the answer, in part, to a question raised by Wall in §9 of [3], his basic paper on nonsimply connected surgery. To explain this, let X be a finite Poincaré complex of formal dimension m, and let ν be a vector bundle over X of the fiber homotopy type of the "Spivak normal fibration." In §3 of [3] Wall defines a cobordism group $\Omega_m(X, \nu)$ based on degree 1 maps $\phi: M\to X$ and framings of $T(M)\oplus\phi^*\nu$. In §5 (for m even) and §6 (for m odd) Wall defines a covariant functor L_m from finitely presented groups to abelian groups and a map $\theta:\Omega_m(X, \nu)\to L_m(\pi_1 X)$ which describes the obstruction to surgering ϕ: $\theta(\alpha)=0$ if and only if α contains a simple homotopy equivalence $\phi: M\to X$. L_m and L_{m+4} are the same by definition. To give a geometric expression to this periodicity, in §9 Wall defines a pairing

$$L_m(\pi)\otimes\Omega_n\to L_{n+m}(\pi)$$

by associating, to N^n and the map $\phi: M\to X$, the product $\phi\times\mathrm{id}$: $M\times N^n\to X\times N^n$. This makes $L_*(\pi)$ into an Ω^*-module and Wall shows that the action of $[CP_2]$ is the periodicity identity $L_m=L_{m+4}$; Wall then conjectures that the action of $[N]$ depends only on the index $I(N)$. Here we show that this is true, at least for m odd and n even.

THEOREM 1. *For m odd and $n=2r$, the pairing $L_m(\pi)\otimes\Omega_n\to L_{n+m}(\pi)$ sends $\alpha\otimes[N]\to I(N)\alpha$ for r even, $\alpha\otimes[N]\to 0$ for r odd.*

The case $m=2k$ appears to be easier to handle, since the obstruction is the intersection form, which is just the $\otimes$-product of the form on M and the form on N, and is homologically defined. The self intersection form does introduce a complication, at least if k is odd. In any case we concentrate here on $m=2k+1$.

2. We freely use here terms and notation introduced by Wall in [3, §5, §6 mostly]. Throughout π will be a fixed finitely presented group and $\Lambda=\mathbf{Z}[\pi]$. Let (K, λ, μ) be a standard kernel, as in Wall's §5. So K is a free Λ-module (of finite dimension) and $K=S_1\oplus S_2$ where S_1 has a specified basis $e_1, \cdots, e_\nu$ and S_2 has basis $f_1, \cdots, f_\nu$. λ is a $(-1)^k$-conjugate symmetric quadratic form on K (briefly,

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