SMOOTH HOMOTOPY PROJECTIVE SPACES

BY CHARLES H. GIFFEN¹

Communicated by William Browder, September 19, 1968

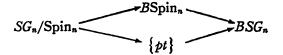
Introduction. In [5] we considered certain fixed point free involutions on Brieskorn manifolds as weakly complex bordism elements. In [4] we considered associated examples of smooth normal invariants for real projective spaces, settling the realizability question for dimensions $\neq 1 \mod 4$ and the desuspendability question for dimensions 4k+1. The object of this study is the classification of these smooth normal invariants given by the Brieskorn examples. Our results overlap somewhat with Atiyah and Bott [2] as well as Browder [3], but our methods are entirely different and our results rather more refined. Full details of these and related results will appear elsewhere.

1. Smooth normal invariants. Following Sullivan [6], we regard a smooth normal invariant of a space X as an element of [X, G/O]. Of course, we have $G/O \cong SG/SO$. We need the fibers SG/Spin of $BSpin \rightarrow BSG$ and $SO/Spin \simeq P^{\infty}$ of $BSpin \rightarrow BSO$. The spaces SG/SO, SG/Spin, SO/Spin have their Whitney H-space structures under which the sequence

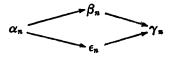
$$SO/Spin \rightarrow SG/Spin \rightarrow SG/SO$$

is a multiplicative fibration.

A map $\mu: SG/Spin \rightarrow BO$ is constructed as follows. Let γ_n denote the universal fiber space over BSG_n with fiber S^{n-1} , β_n the pullback to $BSpin_n$, and α_n the pullback to $SG_n/Spin_n$; also, let ϵ_n denote the S^{n-1} fibration over a point. Corresponding to the commutative diagram



of spaces, there is the commutative diagram



¹ Research supported by NSF grant GP-6567.