

THE PSEUDO-CIRCLE IS UNIQUE¹

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1. **Introduction.** A well-known and hitherto unsolved problem concerning circularly chainable continua is the question raised by R. H. Bing in 1951 in [3, p. 49] of whether or not each two planar non-snake-like hereditarily indecomposable circularly chainable continua, described in [3, p. 48], are homeomorphic. Such continua have subsequently been referred to in the literature as "pseudo-circles" and have been discussed by F. B. Jones at the Summer Institute on Set-Theoretic Topology, University of Wisconsin, 1955 [7]. The purpose of this paper is to give an affirmative answer to this question of whether or not the pseudo-circle is topologically unique.

2. **Preliminaries.** The concepts of r -pattern, cyclic r -pattern, linear representation, and associated types of refinements involving p -chains and circular p -chains were introduced by this author in [4] and [5]. In this present paper we use the terms and some of the results of [4] and [5] together with the following extensions of these concepts. If f is a cyclic r -pattern, then the linear representation of f having the same domain as f is said to be the *canonical linear representation* of f . The union of all linear representations of a cyclic r -pattern f is defined to be the *universal linear representation* of f . The *length of a p -chain or circular p -chain* is defined to be equal to the number of links of the p -chain or circular p -chain, respectively, and the *length of a pattern* is defined to be equal to the number of elements in the range of the pattern. If a p -chain P is a refinement of a p -chain Q then the minimum of the lengths of the maximal sub- p -chains of P that are refinements of single-link sub- p -chains of Q is said to be the *rank* of P in Q . It is observed that in the case that P or Q are circular p -chains, the term sub- p -chain refers to a subcollection of the links of P or Q that are contiguous with respect to the cyclic ordering of links. In this development the various types of linear representations have roles analogous to those of transformations between covering spaces. The

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