RECTIFIABILITY AND INTEGRALGEOMETRIC MEASURES IN HOMOGENEOUS SPACES[1]

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1. Introduction. In recent years much of the progress which has been made in geometric measure theory has depended on knowledge of the geometric structure of subsets of n dimensional Euclidean space $\boldsymbol{R}^n$ relative to some measure such as the k dimensional Hausdorff measure H^k, $0<k<n$. For example, the proof in [5] of the existence of solutions for the least area problem (Plateau's problem) and the minimal surface problem depends essentially on this structure theory.

Central to the structure theory is the characterization of rectifiable subsets in terms of their projection properties. Such results were obtained first by Besicovitch in [1] for H^1 in $\boldsymbol{R}^2$, then by Federer in [4] for general measures on $\boldsymbol{R}^n$. In the present paper we give global generalizations of these theorems to measures on a separable manifold X with a transitive group of diffeomorphisms G. The proofs will appear in [3].

Turning to integralgeometric (Favard) measure, we recall that it is easy to determine the relationship between H^k and the classical integralgeometric measure on $\boldsymbol{R}^n$ once the projection properties of sets of finite H^k measure are known and Crofton's formula is verified for k rectifiable sets. We define generalized integralgeometric measures in a manifold of constant curvature and apply our structure theory, together with the integralgeometric formulas of [2], to determine the relationship between H^k and these measures.

2. Preliminaries. Recall that in a separable Riemannian manifold the statements "$H^k(A)=0$" and "A is H^k sigma finite" are independent of the metric; hence are meaningful for $A\subset X$.

Let $\dim X=n$.

Denote $\alpha(n)=H^n(\boldsymbol{R}^n\cap\{x: |x|<1\})$ and

$$\beta(n, k) = \binom{n}{k}^{-1} \alpha(n)^{-1}\alpha(k)\alpha(n-k).$$

Let ϕ be a nonnegative measure on X such that closed sets are ϕ measurable.

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