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MAXIMAL FUNCTIONS FOR A CLASS OF LOCALLY COMPACT NONCOMPACT GROUPS

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In this note, we briefly describe some maximal theorem results to be proved in detail in an appendix (§4) to the paper [PT]. In [PT], maximal averages taken over sets of unbounded measure for functions of several variables over a local field are used to study singular integrals. The results on maximal functions can, however, be obtained for a large class of topological groups, and it is these results which we will describe. The results generalize theorems on maximal functions appearing in [EH], where the sets over which averages are taken have bounded measures. Let Z denote the integers. Our hypothesis is that G is a locally compact group (written multiplicatively) with left Haar measure λ and that $\{U_n : n \in Z\}$ is a neighborhood base at the identity e consisting of relatively compact Borel sets satisfying

$$(1)\quad \begin{aligned} &\text{(i) } U_{n+1} \subsetneq U_n \text{ for all } n \in Z \text{ and } \lim_{n\to-\infty} \lambda(U_n) = \infty;\\ &\text{(ii) } \lambda(U_n U_n^{-1}) < C\lambda(U_n),\ C \text{ constant},\ n \in Z;\\ &\text{(iii) For each } n \in Z \text{ there is an } l(n) \in Z \text{ such that } U_{l(n)} \supset U_n^{-1} U_n\\ &\quad \text{and } U_j \not\supset U_n^{-1} U_n \text{ if } j > l(n). \text{ And, there is a constant } \alpha \text{ such}\\ &\quad \text{that } \lambda(U_{l(n)}) < \alpha\lambda(U_n) \text{ for all } n \in Z. \end{aligned}$$

For such an "M-sequence," we can prove [PT] the following theorem.