ON SINGULARITIES OF SURFACES IN E^4

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1. **Notation.** Let f: $M^2 \to E^4$ be an immersion of a compact orientable surface. Let $e_1e_2e_3e_4$ be orthonormal righthanded frames, e_1e_2 tangent and agreeing with a fixed orientation of M. As usual define ω_i and ω_{ij} by

$$df = \sum \omega_i e_i \qquad de_i = \sum \omega_{ij} e_j, \quad i = 1, \cdots, 4.$$

The connection forms in the tangent and normal bundles are respectively ω_{12} and ω_{34}. The respective curvature forms are $d\omega_{12}$ and $d\omega_{34}$. The Gauss curvature K and the normal curvature N satisfy (and may be defined by)

$$d\omega_{12} = - K\omega_1 \wedge \omega_2, \qquad d\omega_{34} = - N\omega_1 \wedge \omega_2.$$

2. **Statement of the main results.**

THEOREM 1. *Suppose f: $M \to E^4$ is an immersion such that N is everywhere positive (negative). Then*

$$\chi(NM) = - 2\chi(M) \qquad (\chi(NM) = 2\chi(M)).$$

Here $\chi(NM)$ is the Euler characteristic of the normal bundle and $\chi(M)$ is the Euler characteristic of M.

COROLLARY 2. *Every immersion of the sphere or torus must have a point where $N=0$.*

The proof of Theorem 1 uses a geometrically defined field of tangent axes. In order to define these axes we review some of the local theory of surfaces in E^4.

3. **The curvature ellipse** [1]. The local invariants of a surface in E^4 are characterized by an ellipse in the normal plane. To define this ellipse let us first define a map η: $S_p \to N_p$, S_p is the unit tangent circle at p and N_p is the normal plane at p. Let $\gamma(s)$ be a geodesic of M through p such that $d\gamma/ds(p) = e_1$, where e_1 is a unit vector at p. Define η by $\eta(e_1) = d^2\gamma/ds^2(p)$. The curvature ellipse is the image of S_p under η.

The mean curvature vector $\mathcal{H}$ is the position vector of the center of this ellipse.