

ON THE LAWS OF CERTAIN LINEAR GROUPS

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In recent years a great deal of attention has been devoted to the study of finite nonabelian simple groups, but one aspect which seems to have been little considered is that of the laws which they satisfy. In a recent paper, two of the present authors gave a basis for the laws of $\text{PSL}(2, 5)$ (Cossey and Macdonald [1]), and here we present bases for the laws of $\text{PSL}(2, 7)$ and $\text{PSL}(2, 9)$. An important feature of the proofs of these results is the knowledge of a two variable basis for the laws of S_4 (the symmetric group of degree 4), which we also state. We show how bases for certain $\text{SL}(2, p)$ can be derived from bases for the corresponding $\text{PSL}(2, p)$. Finally, we give some laws which hold in $\text{PSL}(2, p^n)$ (p a prime) in more general cases.

In notation and terminology we follow the book of Hanna Neumann [2]. We would draw the reader's attention particularly to the law v_n and its properties, given in 52.31 and 52.32 of [2]. Also note that $\text{var } G$ denotes the variety generated by a group G .

Our results are as follows.

THEOREM 1. *A basis for the laws of $\text{var } S_4$ is*

- (1) $x^{12} = 1$,
- (2) $[x^6, y^6] = 1$,
- (3) $[x, y]^6 = 1$,
- (4) $[x^2, y^2]^2 = 1$,
- (5) $[x^3, y^3, y^3]^3 = 1$,
- (6) $[x, y]^3, y^3, y^2] = 1$.

THEOREM 2. *A basis for the laws of $\text{var } \text{PSL}(2, 7)$ is*

- (1) $x^{84} = 1$,
- (2) $\{ (x^{21}y^{21})^{52}(x^{21}y^{63})^4[x^{21}, y^{42}]^9 \}^{21} = 1$,
- (3) $\{ (x^{28}y^{28})^{57}[x^{28}, y^{28}]^{14} \}^{28} = 1$,
- (4) $[x^3, y^{12}]^{12} = 1$,
- (5) $\{ [y^{-24}x^{21}y^{24}, y^{-12}x^{21}y^{12}]^{37}, x^{21}]^{13}y^{-48}[x^{21}, y^{48}]^7y^{48} \}^{21} = 1$,
- (6) $\{ [y^{-12}x^{28}y^{12}, x^{28}]^6[x^{28}, y^{36}]^2 \}^{28} = 1$,
- (7) $\{ (x^7y^7)^{12}(x^7y^{49})^{12}[x^{35}, y^{49}]^{11} \}^{42} = 1$,
- (8) $\{ (x^{77}y^{77})^{72}[x^7, y^7]^{15}[x^7, y^{77}]^3 \}^{36}[x^{42}, y^{42}]^6 \}^7 = 1$,

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