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CLASSIFICATION OF KNOTS IN CODIMENSION TWO

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Communicated by W. Browder, May 31, 1968

Introduction. In this paper we consider smooth knots, i.e., smooth embeddings $\phi: S^n \rightarrow S^{n+2}$, $n \geq 3$. Two knots ϕ and η are said to be equivalent if there is a diffeomorphism $f: S^{n+2} \rightarrow S^{n+2}$ such that $f\phi(S^n) = \eta(S^n)$. The embedding ϕ extends to an embedding $\bar{\phi}: S^n \times D^2 \rightarrow S^{n+2}$, and any two such extensions are ambient isotopic relative to $S^n \times 0$. Hence if $A = \text{cl}(S^{n+2} - \bar{\phi}(S^n \times D^2))$, the pair $(A, \partial A)$ is determined up to diffeomorphism by the equivalence class of ϕ . We call $(A, \partial A)$ the complementary pair, or simply the complement, of the knot ϕ . In this paper we show that if $\pi_1 A$, the fundamental group of the knot, is infinite cyclic, then there is at most one knot inequivalent to ϕ with complementary pair $(B, \partial B)$ of the same homotopy type as $(A, \partial A)$. This result is of interest because for any $n \geq 3$ there are many inequivalent knots $\phi: S^n \rightarrow S^{n+2}$ with fundamental group \mathbb{Z} , see for example [12]. (The result also holds in the P.L. case, provided ϕ extends to a P.L.-embedding $\bar{\phi}: S^n \times D^2 \rightarrow S^{n+2}$.)

1. Knots with diffeomorphic complements. In [4], Gluck showed that homeomorphisms of $S^2 \times S^1$ are isotopic if and only if they are homotopic and used this result to conclude that there are at most two knots $\phi: S^2 \rightarrow S^4$ with homeomorphic exteriors. In [1], W. Browder studied the pseudo-isotopy classes of diffeomorphisms (and P.L. equivalences) of $S^1 \times S^n$ for $n \geq 5$. He showed that two P.L. equivalences are pseudo-isotopic if and only if they are homotopic. For the group $\mathfrak{D}(S^n \times S^1)$ of pseudo-isotopy classes of diffeomorphisms, he obtained the exact sequence