THE SECOND HOMOTOPY GROUP OF SPUN 2-SPHERES IN 4-SPACE

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1. **Introduction.** Andrews and Curtis [1] have shown that the second homotopy group of the complementary domain of a locally flat 2-sphere S^2 in the 4-sphere S^4 may not be trivial. This was shown to be the case if S^2 is formed by spinning the trefoil knot. Epstein [3] has shown that if S^2 is a spun nontrivial 2-sphere, then $\pi_2(S^4-S^2)$ is a free abelian group of infinite rank. Fox [6] has suggested that it might be more fruitful to consider the second homotopy group with its π_1-action, and has asked for an algorithm for calculating $\pi_2(S^4-S^2)$ as a $J\pi_1$-module. Sumners [8] has constructed a knotted 2-sphere in S^4 for which π_2 has nontrivial $J\pi_1$-torsion.

The following theorem gives the structure of π_2 as a $J\pi_1$-module for the case of spun 2-spheres.

THEOREM 2. *If $k(S^2)\subset S^4$ is a 2-sphere formed by spinning an arc A about the sphere S^2 and $(x_0, x_1, \cdots, x_n: r_1, r_2, \cdots, r_m)^\phi$ is a presentation of $\pi_1(S^4-k(S^2))$ with x_0 the image of the generator of $\pi_1(S^2-A)$ under the inclusion map, then*

$$\left(X_i(1 \leqq i \leqq n): \sum_{i=1}^{n} (\partial r_j/\partial x_i)*X_i = 0 \ (1 \leqq j \leqq m)\right)$$

is a presentation of $\pi_2(S^4-k(S^2))$ as a $J\pi_1$-module.

2. **Outline of proof.** Let S^n be the standard n-sphere. Let $S^n_\pm$ be the closed domains of S^n-S^{n-1}. Let A be an arc in S^3_+ which meets S^2 only in the end-points of A. Now rotate S^3_+ about S^2. Then A sweeps out a 2-sphere $k(S^2)$ called a spun 2-sphere [2].

THEOREM 1. *If $k(S^2)\subset S^4$ is a spun 2-sphere, then $\pi_2(S^4-k(S^2)) \simeq K/[K,K]$, where K is the kernel of the homomorphism $i_*: \pi_1(S^3-k(S^2)) \to \pi_1(S^4-k(S^2))$ induced by inclusion and $[K, K]$ is the commutator subgroup of K.*

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