SQUARING OPERATIONS IN THE ADAMS SPECTRAL SEQUENCE

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It has been long known that the cohomology $H(A)$ of the mod 2 Steenrod algebra A admits squaring operations. (For example, see [6].) Since $H(A)$ is isomorphic to the E_2 term of the mod 2 Adams spectral sequence [2], it is natural to inquire as to the relation of these squaring operations to the structure of the Adams spectral sequence. In this note we announce some results of this type extending those of [5]. Details will appear elsewhere.

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1. The quadratic construction. The main tool in this study is the quadratic construction, a functor from pointed spaces to filtered spaces which has been studied by J. F. Adams, M. G. Barratt and M. Mahowald (unpublished).

Let X be a finite CW complex with basepoint. Denote by $Q'^n(X)$ the space $S^n \ltimes (X \wedge X)$. (If B is a space with basepoint b, $A \ltimes B$ shall mean $A \quad B/A \times b$.) Define the involution $T: Q'^n(X) \to Q'^n(X)$ by $T(x, y, z) = (-x, z, y)$, where $-x$ is the point antipodal to x. This defines an action of Z_2 on $Q'^n(X)$ and we set $Q^n(X) = Q'^n(X)/Z_2$. $Q(X) = Q^\infty(X)$ is called the quadratic construction on X and is clearly a functor. It is not difficult to show that $Q^r(S^n)$ is homeomorphic to $S^n \wedge P_n^{n+r}$, where P_n^{n+r} denotes the stunted real projective space RP^{n+r}/RP^{n-1}.

LEMMA (1.1). *If* $n = 2^q$ *where* $q \geqq \phi(r)$ [3], $S^n = P_n^n$ *is a retract of* P_n^{n+r}.

PROOF. [3], [4].

Denote by $\phi_{n,r}$ such a retraction.

Let $\bar{\alpha} \in \pi_t^s(S^0) = G_t$ correspond to $\alpha \in E_2^{s,s+t}(S^0)$ and let $\bar{\alpha}$ be represented by a map $f: S^{n+t} \to S^n$ where $n = 2^q$ and $q \geqq \phi(r)$ (and perhaps much larger). Define $\Phi_r(f): S^{n+t} \wedge P_{n+t}^{n+t+r} \to S^{2n}$ by $\Phi_r(f) = (S^n \wedge \phi_{n,r}) \circ Q^r(f)$. Let

$$\theta_k: (E^{2(n+t)+k}, S^{2(n+t)+k-1}) \to (S^{n+t} \wedge P_{n+t}^{n+t+k}, S^{n+t} \wedge P_{n+t}^{n+t+k-1})$$