RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited. Manuscripts more than eight typewritten double spaced pages long will not be considered as acceptable. All papers to be communicated by a Council member should be sent directly to M. H. Protter, Department of Mathematics, University of California, Berkeley, California 94720.

SPHERE-PACKING IN THE HAMMING METRIC[1]

BY ROBERT J. MCELIECE AND HOWARD RUMSEY, JR.

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Let $V_n(2)$ be the n-dimensional vector space over GF(2), with vectors represented as n-tuples of 0's and 1's. The *Hamming metric* $d(x, y)$ is defined to be the number of coordinates in which x and y disagree. If $A = \{a_1, a_2, \cdots, a_M\}$ is a set of M vectors, we define $d(A) = \min_{i \neq j} d(a_i, a_j)$, and $\bar{d}(A) = \operatorname{mean}_{i \neq j} d(a_i, a_j)$. Finally define

$$D(n, M) = \max_{|A| = M} d(A).$$

We present in this paper a method of obtaining an upper bound on $D(n, M)$ which is always at least as good as the well-known bounds, and which is frequently better. At the same time, the method gives a satisfactory explanation of the relationship between the various known upper bounds on $D(n, M)$ (Hamming [1], Plotkin [1], and Elias [2]). The weakness of the method seems to be that for the most part it deals only with the average distance between vectors, and further progress probably awaits a technique which is able to deal more directly with the minimum distance.

We need three theorems. Throughout $A = \{a_1, a_2, \cdots, a_M\}$ is a set of M vectors from $V_n(2)$.

THEOREM 1. *Let $S_r(x)$ be the sphere of radius r centered at x. Then the mean value of $|S_r(x) \cap A|$ as x varies over $V_n(2)$ is*

$$M_r = \frac{M}{2^n} \sum_{k \leq r} \binom{n}{k}.$$

[1] This paper presents the results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under Contract No. NAS 7-100, sponsored by the National Aeronautics and Space Administration.