Since the factors in the first two sets of brackets are finite Blaschke products and the zero in the third is a convex combination of such, and since the coefficients are nonnegative and sum to 1, the proof is complete.

References

1. C. Caratheodory, Theory of functions. Vol. 2, Chelsea, New York, 1954.

2. R. R. Phelps, Extreme points in functions algebras, Duke Math. J. 32 (1965), 267-278.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

EXACTNESS OF INVERSE LIMITS

BY ULRICH OBERST

Communicated by Saunders Mac Lane, May 20, 1968

I. The problem of this investigation is to characterize those small categories X for which the inverse limit

$$\operatorname{proj}_{\mathbf{X}} \lim AB^{\mathbf{X}} \to AB$$

is exact. Here AB is the category of abelian groups, and AB^{x} is the category of functors from X to AB. In this context I conjecture the following

THEOREM I. Let X be a small category. Then the following assertions are equivalent:

(1) The inverse limit proj $\lim_{\mathbf{X}} : AB^{\mathbf{X}} \rightarrow AB$ is exact.

(2) For every abelian category \mathfrak{A} with exact direct products, the inverse limit proj $\lim_{x} \mathfrak{A}^{x} \to \mathfrak{A}$ is exact.

(3) Every connected component Y of X contains an object y together with an endomorphism $e \in Y(y, y)$ such that the following properties are satisfied:

(i) y is a smallest object of Y, i.e., for any object $z \in Y$ there is a morphism $y \rightarrow z$.

(ii) e equalizes any two morphisms with the same codomain and domain y, i.e., any diagram $y \xrightarrow{a} y \xrightarrow{a} z$ is commutative.

At present, I can prove the equivalence of (1) and (2) and the implication $(3) \rightrightarrows (1)$ in general, i.e., without any additional condition on X. The implication $(1) \rightrightarrows (3)$ holds at least if one of the following conditions on X is satisfied: