TOPOLOGICAL EMBEDDINGS IN CODIMENSION ONE[1]

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1. **Introduction.** Suppose Q^{n+1} is a piecewise linear $(n+1)$-manifold and M^n is a closed topological n-manifold embedded in int Q^{n+1}. We seek conditions on the embedding of M which insure that M has arbitrarily small neighborhoods which look like regular neighborhoods of a piecewise linear (PL) submanifold of Q. In particular, we would like M to be contained in a compact $(n+1)$-dimensional PL submanifold N of Q such that

(1) $M \subset \text{int } N$,

(2) M is a strong deformation retract of N, and

(3) $N-M$ is PL homeomorphic to bd $N \times [0, 1)$.

We call any compact (connected) PL submanifold N of Q satisfying (1) a PL *manifold neighborhood of* M.

We say that $Q-M$ *is* 1-*lc at* M if for each open set U containing M there is an open set V, $M \subset V \subset U$, such that each loop in $V-M$ is null homotopic in $U-M$. The purpose of this note is to show that, if M is simply connected and $n \geqq 5$, then M has PL manifold neighborhoods satisfying (2) and (3) above if and only if $Q-M$ is 1-*lc* at M.

All homology and cohomology groups will be singular with Z coefficients. i_* (i^*) will denote an inclusion induced map between homology or homotopy (cohomology) groups. The symbol $\approx$ means is isomorphic to or is PL homeomorphic to, depending on the context. I denotes the unit interval $[0, 1]$.

2. **Statement of results.** Let Q^{n+1} be a connected PL $(n+1)$-manifold, M^n a closed, 1-connected topological n-manifold embedded in int Q. Our main result is

THEOREM 1. *If $n \geqq 5$, there is a closed PL n-manifold M^* such that M has arbitrarily small PL manifold neighborhoods which are PL homeomorphic to $M^* \times I$ and satisfy* (2) *and* (3) *above if and only if $Q-M$ is* 1-*lc at* M.

The proof is postponed until §3.

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