REGULAR NEIGHBORHOODS ARE NOT TOPOLOGICALLY INVARIANT

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In [5], Siebenmann and Sondow have shown that there exist topologically equivalent $\mathrm{PL}(n+3, n+1)$-sphere pairs for $n \geqq 2$ that are combinatorially distinct. In this note, combining their analysis of strong h-cobordisms of certain higher dimensional knots and isoneighboring theorem due to Noguchi [3] and [4], we show the following:

THEOREM. *Assume $n = \text{even} \geqq 2$. Then there exist infinitely many combinatorially distinct* $\mathrm{PL}(n+3, n+1)$*-manifold pairs* (V_k, K_k), $k = 1, 2, \cdots$, *that are not abstract regular neighborhoods but topologically equivalent to an abstract regular neighborhood* (V_0, K_0).

REMARK. Each submanifold K_k is a $\mathrm{PL}(n+1)$-sphere which is 1-flat in V_k with only one singularity. (For 1-flat embeddings and singularities, see [3].)

An implication of the Theorem is that regular neighborhoods are not topologically invariant. More explicitly we may say:

COROLLARY. *The collapsing is not topologically invariant.*

We note here that (V_k, K_k) and (V_0, K_0) have the vanishing Whitehead torsion, since K_0 is simply connected. However, in the subsequent paper [2], we shall show that the topological invariance of Whitehead torsions is equivalent to that of regular neighborhoods of polyhedra in the sufficiently high-dimensional euclidean space.

1. The construction. In the following, we shall use the notations in [5]. However, we shall be concerned mainly with the combinatorial (or PL) objects. By a PL *n-knot* we shall mean a $\mathrm{PL}(n+2, n)$-sphere pair (S^{n+2}, L^n) such that L^n has a collar neighborhood $(L^n \times D^2)$ in S^{n+2} [1] and [4].

LEMMA 1. *Assume $n = \text{even} \geqq 2$. Then there exist infinitely many invertible strong h-cobordisms of* PL *n-knots*

$c_k = ((W_k, M_k); (S_0, L_0), (S_k, L_k))$, $k = 1, 2, \cdots$, *such that*

(1) (S_k, L_k) *and* (S_0, L_0) *are combinatorially equivalent,*

(2) $\pi_1(S_0 - L_0) \cong J \times G$, *where J and G are the infinite cyclic group and the binary icosahedral group, respectively, and*