SOME RESULTS IN THE THEORY OF H-SPACES

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1. We study H-spaces which are connected finite complexes, with $H_*(X, Z)$ free of 2-torsion, by considering the complex K-theory of their projective planes. In particular we show that no 7 sphere-bundle over an 11 or 15 sphere can be an H-space,[1] and that a homotopy associative, homotopy commutative H-space must have the homotopy type of a torus.

2. Let X be an H-space as above, then $H^*(X, Q)$ is an exterior algebra on odd dimensional generators, and we assume that the Hopf algebra structure, with comultiplication induced by the H-space map, is primitively generated. The number of generators we call the rank of X and the dimensions in which they occur, the type of X.

THEOREM 1. *Let $H^*(X, Q)$ have not more than one generator in any one dimension, then*

(i) *If the rank of X is* 1, *its type is* 1, 3 *or* 7.

(ii) *If the rank of X is* 2, *its type is* (1, 3) (1, 7) (3, 5) *or* (3, 7).

(iii) *If the type of X contains* $4m-1$, *with m not divisible by* 4, *then its type contains* $4q-1$, *for all q with* $2 \leqq q \leqq m$.

(iv) *If the type of X contains* $2m-1$, *with m odd, then it contains* $2q-1$, *for all q with* $2 \leqq q \leqq m$.

Part (i) is well known. It was first proved in [**3**] and we have generalized the proof of [**5**]. Part (ii) answers questions raised in [**4**] and [**8**]. It was the solution of this problem that led to the rest of this work. In general, if you specify the number of generators, better results can be obtained by applying the methods of this paper rather than the results.

3. Let X be a noncontractible connected finite complex which is a homotopy commutative H-space, then $H_*(X, Z)$ is free of 2-torsion by Theorem 8.5 of [**7**].

THEOREM 2. *Let X be a homotopy associative, homotopy commutative H-space, then X has the homotopy type of a torus.*

[1] *Added in proof.* Dr. R. Douglas and Dr. F. Sigrist have recently announced their independent proof of this result.