THE COHOMOLOGICAL DIMENSION OF STONE SPACES

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The purpose of this note is to announce a few inequalities involving the cohomological (sheaf-theoretic) dimension of locally compact, totally disconnected Hausdorff spaces, herein called *Stone spaces*. Throughout, R will denote a commutative regular ring with maximal ideal space X. (Then X is compact and totally disconnected.) For each ideal J in R let $U[J]$ denote the corresponding open subset of X, and for each R-module A, let $\mathfrak{A}(A)$ denote the corresponding sheaf of modules, as defined in [2].

THEOREM 1. $\operatorname{Ext}^n_R(J, A)$ *and* $H^n(U[J]; \mathfrak{A}(A))$ *are naturally isomorphic.*

THEOREM 2. *Let* $\mathfrak{F}$ *be a sheaf over the Stone space* X, *and let* $\mathfrak{U}$ *be a covering of* X *consisting of compact open sets. Then the natural maps* $H^n(\mathfrak{U}; \mathfrak{F}) \to \check{H}^n(X; \mathfrak{F}) \to H^n(X; \mathfrak{F})$ *are all isomorphisms.*

Let $\dim X$ denote the cohomological dimension of X, and cov dim X the covering dimension of X, based on arbitrary (not necessarily finite) open coverings. (It is not hard to show that for Stone spaces, cov dim $X \leqq n$ iff X has a compact open cover of order n.) Finally, let $h\cdot\dim_R J$ denote the homological (projective) dimension of the ideal J.

COROLLARY. $h\cdot\dim_R J \leqq \dim U[J] \leqq \operatorname{cov\,dim} U[J]$.

Since the only projective R-modules are direct sums of principal ideals [1], we see that $h\cdot\dim_R J=0$ iff cov dim $U[J]=0$, and, by the corollary, iff $\dim U[J]=0$. In order to see that equality need not always hold in the corollary, let us define the *rank* ρ of a space X by agreeing that $\rho(X) \leqq n$ iff X can be written as a union of $\aleph_n$ (or fewer) compact sets.

THEOREM 3. *For any Stone space* X, $\dim X \leqq \rho(X)$.

EXAMPLE 1. Let Ω be the set of countable ordinals, with the order topology. Then $\dim \Omega = 1$, but cov dim $\Omega = \infty$. (The second assertion may be verified directly; the first then follows from Theorem 3 and the remarks following the corollary.)

The next example shows that the inequality in Theorem 3 cannot be sharpened.