ON SOLUTIONS OF NTH ORDER LINEAR DIFFERENTIAL EQUATIONS WITH N ZEROS

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In a very interesting paper in 1958, Hartmann proved the following result [2]: The equation

$$L_n y = y^{(n)} + p_{n-1}(x)y^{(n-1)} + \cdots + p_0 y = 0, \tag{1}$$

with $p_i \in C(a, b)$, has a nontrivial solution with n zeros, each zero being counted in accordance with its multiplicity, if and only if there is a solution with n distinct zeros on (a, b).

Left as an open question in that paper was whether the result remains true when the restriction that the interval be open is removed. The answer to that question is yes if the interval is half-open and no if it is closed. In addition, there is the problem of giving a more exact description to the solution with distinct zeros, that is, does there exist a solution with exactly n distinct zeros all of which are simple? The answer to this question is also yes.

As an application of these results it is established that the first conjugate point $\eta_1(t)$ of the point t is a continuous function of the coefficients in (1). It is, by the way, not difficult using the techniques in [**3**] to establish that $\eta_1(t)$ is a continuous function of t; however, an easier proof is available using the results stated herein. The results of this paper are stated in terms of equation (1) but could just as easily be stated for a more general equation as in [**4**].

Results. By a solution it is understood that what is meant is a nontrivial solution.

DEFINITION 1. Let $\eta_1(a)$, the first conjugate point of a, be the smallest $b > a$ such that there is a solution of (1) with n zeros on $[a, b]$.

THEOREM 1. *The equation* (1) *has a solution with n zeros on* $[a, b)$ *if and only if there is a solution of* (1) *with a simple zero at a, whose first n zeros on* $[a, b)$ *are simple zeros.*

In order to establish this result a number of lemmas are required some of which are of interest in their own right. The most important of these lemmas is stated below.

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