GLOBAL SOLUTIONS OF HYPERBOLIC SYSTEMS OF CONSERVATION LAWS IN TWO DEPENDENT VARIABLES

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We are interested in general hyperbolic systems of the form

$$(1) \qquad u_t + f(u, v)_x = 0, \qquad v_t + g(u, v)_x = 0$$

with initial data

$$(2) \qquad (v(0, x), u(0, x)) = (v_0(x), u_0(x)).$$

The vector $U = (v, u)$ is a function of t and x, $t \geqq 0$, $-\infty < x < \infty$, and the functions f and g are C^2 functions of two real variables. We assume that the system (1) is hyperbolic in some open set $\mathcal{U}$ in the v-u plane, with $f_v g_u > 0$. Let $DF(U)$ and $D^2F(U)$ denote respectively the first and second Fréchet derivatives (see [2]) of the vector function $F = (f, g): \mathcal{U} \to R^2$; and let $r_j(U)$, $j = 1, 2$, be the eigenvectors of $DF(U)$, with orthogonal vectors $l_j(U)$, $j = 1, 2$: $l_i(U)r_j(U) = 0$ for $i \neq j$.

THEOREM 1. *Let the system* (1) *be hyperbolic in an open set* $\mathcal{U}$ *in the* v-u *plane. Then* (a) *the system* (1) *is genuinely nonlinear in the jth characteristic field at* $U \in \mathcal{U}$ *(see Lax* [6]) *if and only if*

$$l_j(U) D^2F(U)[r_j(U), r_j(U)] \neq 0;$$

(b) *the system* (1) *satisfies the Glimm-Lax shock interaction condition (condition* (c) *of* [4]) *in* $\mathcal{U}$ *provided that left eigenvectors* $l_j(U)$ *can be chosen so that*

$$l_j(U) D^2F(U)[r_k(U), r_k(U)] > 0, \qquad j, k = 1, 2, \ j \neq k, \ U \in \mathcal{U}.$$

The Glimm-Lax shock interaction condition states that the interaction of two shocks of one family produces a shock of the same family and a rarefaction wave of the opposite family. Moreover, for sufficiently weak shocks, we are able to prove an analogous theorem for $n \times n$ systems of conservation laws, $n \geqq 2$, which locally admit Riemann invariants. The proof of it uses some ideas in [3].

We assume that the system (1) is genuinely nonlinear in $\mathcal{U}$, and we normalize r_j by $D\lambda_j(U)[r_j(U)] > 0$, $j = 1, 2$, where $\lambda_j = \lambda_j(U)$ is the eigenvalue asociated with r_j, $\lambda_2 > \lambda_1$. We then normalize l_j by $l_j r_j > 0$,

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