SECOND ORDER LINEAR OSCILLATION WITH INTEGRABLE COEFFICIENTS[1]

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We are here concerned with the oscillatory behaviour of solutions of the following second order ordinary differential equation:

$$x'' + a(t)x = 0, \qquad (' = d/dt,\ t \geqq 0), \tag{1}$$

where $a(t)$ is locally integrable in t. We call equation (1) oscillatory if all solutions of (1) have arbitrarily large zeros, otherwise, we say equation (1) is nonoscillatory. As a consequence of Sturm's Separation Theorem [1], if one of the solutions of (1) is oscillatory, then all of them are. The same is true for nonoscillatory solutions of (1).

Our primary interest here is to consider equations with integrable coefficients, namely those functions $a(t)$ satisfying

$$\lim_{T\to\infty} \int_0^T a(t)dt < \infty. \tag{2}$$

In this case, one may introduce the function $A(t)$ defined by

$$A(t) = \int_t^\infty a(s)ds, \qquad t \geqq 0. \tag{3}$$

In contrast to most of the earlier works on this subject, the coefficient $a(t)$ and its iterated integral $A(t)$ are not assumed to be nonnegative here. We shall be interested in those cases when the iterated integral $A(t)$ is again integrable as a function of t. A typical example of such is the function $a(t) = \sin \beta t/t$, $\beta \neq 0$. As far as we know, the range of β for which equation (1) is oscillatory or nonoscillatory has not been completely determined. There are two exceptions which offer partial solutions. Using Hartman [1, p. 369, Exercise 7.9], one can deduce that (1) is nonoscillatory if $|\beta| > 4$. Willett [3] recently gives results which will yield oscillation of (1) if $|\beta| < 2^{1/2}$ and nonoscillation if $|\beta| > 2^{1/2}$. It will be clear from the results below that (1) is also nonoscillatory when $|\beta| = 2^{1/2}$. We shall discuss further perturbations of this boundary case in an example below.

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