RESEARCH ANNOUNCEMENTS

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NONEXISTENCE AND UNIQUENESS OF POSITIVE SOLUTIONS OF NONLINEAR EIGENVALUE PROBLEMS[1]

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We consider nonlinear eigenvalue problems of the general form:

$$Lu = F(\lambda, x, u), \qquad x \in D, \tag{1}$$

$$\beta(x)\partial u/\partial\nu + \alpha(x)u = 0, \qquad x \in \partial D. \tag{2}$$

Here $x = (x_1, x_2, \cdots, x_m)$ and

$$\left.\begin{aligned} &L\phi \equiv \sum_{i,j=1}^{m} \partial_i[a_{ij}(x)\partial_j\phi] - a_0(x)\phi, \qquad a_{ij}(x) = a_{ji}(x) \\ &\sum_{i,j=1}^{m} a_{ij}(x)\xi_i\xi_j \geqq a^2 \sum_{i=1}^{m} \xi_i^2, \qquad a^2 > 0; \qquad a_0(x) \geqq 0 \end{aligned}\right\} x \in D;$$

$$\left.\begin{aligned} &\frac{\partial\phi}{\partial\nu} \equiv \sum_{i,j=1}^{m} n_i(x)a_{ij}(x)\partial_j\phi \\ &\alpha(x)\beta(x) \geqq 0, \qquad \alpha(x) \not\equiv 0, \qquad \alpha(x) + \beta(x) > 0 \end{aligned}\right\} x \in \partial D. \tag{3}$$

All coefficients and the derivatives of the $a_{ij}(x)$ are continuous on the appropriate closed sets $\bar{D}$ or ∂D, and the latter is piecewise smooth with exterior unit normal vector $(n_1(x), n_2(x), \cdots, n_m(x))$ at $x \in \partial D$. We first prove a simple but useful result on conditions for the nonexistence of positive solutions of (1)–(2).

THEOREM 1. *Let $F(\lambda, x, z)$ be continuous on $x \in D$, $z > 0$. For any positive continuous function $r(x)$ on $\bar{D}$, let $\mu_1\{r\}$ be the least eigenvalue of*

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