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A CHARACTERIZATION OF BANACH ALGEBRAS WITH APPROXIMATE UNIT

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1. **Introduction.** Let $L^1(R)$ denote the space of all complex valued functions on the real line R which are integrable on R in the sense of Lebesgue. It is well known that $L^1(R)$ forms a Banach algebra where the multiplication is defined by convolution; that is,

$$f * g(x) = \int_R f(t)g(x-t)dt,$$

and the norm of an element is defined by $\|f\| = \int |f(t)|\,dt$. In [4] Rudin showed that every function in $L^1(R)$ is the convolution of two other functions. In other words, every element of the convolution algebra $L^1(R)$ can be factored in $L^1(R)$, although this algebra lacks a unit. Subsequently, Cohen [1] observed that the essential ingredient in Rudin's argument is that $L^1(R)$ has an approximate unit in the sense of the following definition.

DEFINITION. A Banach algebra B is said to have an approximate unit if there exists a real number $C \geqq 1$ and a collection $\{e_\lambda: \lambda \in \Lambda\}$ of elements of B, where the index set Λ is a directed set, such that the following two conditions are satisfied: $\|e_\lambda\| \leqq C$, for each λ, and $\lim e_\lambda x = \lim x e_\lambda = x$, for each $x \in B$. Cohen went on to prove that the factorization theorem still holds in any Banach algebra with approximate unit.

The results in this note stem from the observation that multiple factorization occurs in the sup-norm algebra $C_0(R)$, the space of all complex valued continuous functions on R that vanish at infinity; that is, if $f_1, f_2, \cdots, f_n$ are functions in $C_0(R)$ and $\delta > 0$, then there exist functions $g, h_1, h_2, \cdots, h_n$ in $C_0(R)$ such that

$$(1.1) \qquad f_i = gh_i \quad \text{and} \quad \|f_i - h_i\| < \delta \qquad (i = 1, 2, \cdots, n).$$