GENERALIZATION OF THE BIG PICARD THEOREM

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S. Kobayashi defined a pseudodistance d on a complex manifold in such a manner that it depends only on the complex structure of the complex manifold in question [7]. The definition of the pseudodistance can be extended word for word to a complex space (see [3] for definition of a complex space). Let D be the open unit disk in the complex plane $\mathbf{C}$ and ρ the Poincaré-Bergman metric of D. Given two points p and q of a complex space X, choose the following objects:

(1) points $p=p_0, p_1, \cdots, p_k=q$ of X, and

(2) points $a_1, \cdots, a_k, b_1, \cdots, b_k$ of D and holomorphic mappings $f_1, \cdots, f_k$ from D into X such that $f_i(a_i)=p_{i-1}$ and $f_i(b_i)=p_i$ for $i=1, \cdots, k$. For each choice of points and mappings satisfying (1) and (2), consider the number $\rho(a_1, b_1)+ \cdots +\rho(a_k, b_k)$. Let $d(p, q)$ be the infimum of the numbers obtained in this manner for all possible choices.

It is easy to verify that d is a pseudodistance on X. We shall call a complex space hyperbolic if the pseudodistance d_X is a distance. The concept of a hyperbolic space is a generalization of a Riemann surface of hyperbolic type in the sense that a Riemann surface of hyperbolic type is a hyperbolic space. A hyperbolic space (X, d_X) is said to be complete if for any point p of X and any positive number r, the closed ball of radius r around p is compact.

The purpose of this paper is to generalize the big Picard theorem which states that a holomorphic mapping from the punctured disk into the Riemann sphere $P_1(\mathbf{C})$ minus three points can be extended to a holomorphic mapping from the whole disk into $P_1(\mathbf{C})$. H. Huber extended this theorem to the case where the image space is a domain G of hyperbolic type in a Riemann surface R such that the closure of G in R is compact [4].

THEOREM 1. *Let f be a holomorphic mapping from the punctured disk D^* into a hyperbolic space X. Moreover, assume that the complex space X is compact. Then f can be extended to a holomorphic mapping from the whole disk into X.*

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