

Thirdly it follows easily from these observations and the residual nilpotence of free groups that G is parafree of rank r .

Finally we observe that G is not finitely generated, but that $G/\gamma_2 G$ is free abelian of rank two. Hence G is not free.

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SOLVABLE AND NILPOTENT SUBALGEBRAS OF LIE ALGEBRAS

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1. Introduction. We describe here some results on the structure of a Lie algebra in terms of its nilpotent and solvable subalgebras. Proofs will appear elsewhere.

In the following discussions, F is an arbitrary field, \mathfrak{L} is a (finite dimensional) Lie algebra over F and V is a (finite dimensional) vector space over F .

2. Arbitrary Lie algebras. Let \mathfrak{K} be a set of linear transformations in V such that the Lie algebra generated by \mathfrak{K} over F is nilpotent. Then, as is well known, V has a unique vector space decomposition $V = V_0(\mathfrak{K}) + V_*(\mathfrak{K})$ (direct) where $V_*(\mathfrak{K})$ is \mathfrak{K} -stable, $V_0(\mathfrak{K})$ is \mathfrak{K} -stable and $\mathfrak{K}|_{V_0(\mathfrak{K})}$ consists of nilpotent transformations, and where $V_0(\mathfrak{K})$ is maximal with respect to the latter two properties.

One has the following theorem, in spite of the fact that a nilpotent linear Lie algebra cannot always be triangulized over the algebraic closure of its base field.

THEOREM 1.² *If F is infinite and \mathfrak{K} is a subspace of $\text{Hom}_F(V, V)$ such that the Lie algebra generated by \mathfrak{K} is nilpotent, then there exists N in \mathfrak{K} such that $V_0(N) = V_0(\mathfrak{K})$.*

¹ This research was done while the author was a National Science Foundation Postdoctoral Research Fellow at the University of Bonn.

² This is essentially Lemma 3.5 of [4, pp. 87–88]. The author has heard that an independent forthcoming paper of R. Block contains material close to this theorem.