GENERIC ONE-PARAMETER FAMILIES OF VECTOR FIELDS ON TWO-DIMENSIONAL MANIFOLDS

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Communicated by S. Smale, November 30, 1967

Introduction. This paper is an announcement of a study on the theory of the topological variation of the phase space of one-parameter families of vector fields (ordinary differential equations, flows). This theory, sometimes called bifurcation theory, has been developed since H. Poincaré from several points of view; see, for example, [1], [2], [3], [4], [5]. Here, we will be mainly interested in a collection of one-parameter families of vector fields which has the following properties: (a) it is large with respect to all the families, and (b) its elements exhibit a topological variation which is amenable to simple description.

Collections with properties (a) and (b) are currently called "generic" and were introduced in the qualitative global analysis of differential equations by M. Peixoto [6], S. Smale [8], and I. Kupka [11]. See S. Smale [9] for a thorough monography on this field.

The geometry of the set Σ of structurally stable vector fields and the study of "generic" one-parameter families of vector fields are closely related. A vector field is structurally stable if its phase space does not change topologically under small perturbations; a one-parameter family of vector fields exhibits: the simpler a phase-space topological variation, the larger the intersection it has with Σ, or equivalently, the smaller the intersection it has with its complement —the set of nonstructurally stable vector fields.

The importance of the set of nonstructurally stable vector fields for the study of the topological variation of the phase space of vector fields was noticed by A. Andronov and E. Leontovich. In [12] they defined the concept of first-order structural stability as a possible guide to pursue such study.

In compact two-dimensional manifolds, the only case considered here, the set of first-order structurally stable vector fields is an imbedded Banach manifold of class C^1 and codimension one of the Banach manifold of vector fields; see [13]. This fact, although (as follows from [14]) not sufficient to describe completely the "generic"

[1] This work was done with the partial support of the National Science Foundation, Grant GP-5603.