A UNIFORM GENERALIZED SCHOENFLIES THEOREM

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The generalized Schoenflies theorem of M. Brown [2], [3] can be restated in the following way: If S^{n-1} is the equator of S^n, then any locally flat embedding $f\colon S^{n-1}\to S^n$ can be extended to a homeomorphism $F\colon S^n\to S^n$.

The purpose of this paper is to show that, if $n\geqq 5$, the extension F can be constructed in a controlled manner; in particular, if $f\colon S^{n-1}\to S^n$ is close to the inclusion embedding, then $F\colon S^n\to S^n$ can be chosen to be close to the identity homeomorphism. Consequently if, $f, g\colon S^{n-1}\to S^n$ are locally flat embeddings, $n\geqq 5$, and f is close to g, then there is a homeomorphism $H\colon S^n\to S^n$ which is close to the identity such that $Hf=g$.

Let S^{n-1} denote the unit sphere in E^n, B^n the unit ball, and O the origin. If x, y belong to E^n-O, let $\theta(x, y)$ denote the angle in radians between the line segments Ox and Oy, measured such that $0\leqq\theta(x,y)\leqq\pi$. The distance between x and y under the Euclidean metric will be denoted by dist (x, y). If A is a subset of E^n-O, the angular diameter of A, written θ diam A, is defined to be $\sup_{x,y\in A}\theta(x, y)$. This is significant whenever A lies in a half-space.

Now suppose S is a locally flatly embedded $(n-1)$-sphere in E^n which approximates the standard sphere S^{n-1}. Suppose $\phi\colon S^{n-1}\times[0, 1]\to \mathrm{Cl}(\mathrm{Ext}\, S)$ is a collar on S in $\mathrm{Cl}(\mathrm{Ext}\, S)$. If the collar is small, then the θ-diameter of each fiber $\phi(x\times[0, 1])$ is also small. The object of Lemma 2 is to push the collar outward, leaving S fixed, so that its two boundary components are separated by a round sphere with center at O, and so that the θ-diameter of each fiber remains small. The precise statement is as follows.

LEMMA 2. *If $f\colon S^{n-1}\to E^n$, $n\geqq 5$, is a locally flat embedding such that for all $x\in S^{n-1}$, $\theta(x, f(x))<\epsilon$, where $\epsilon<\pi/7$, then there is an embedding $F\colon S^{n-1}\times[0, 1]\to \mathrm{Cl}(\mathrm{Ext}\, f(S^{n-1}))$ such that:*

(1) *$F(x, 0)=f(x)$,*

(2) *$F(S^{n-1}\times 0)$ and $F(S^{n-1}\times 1)$ are separated by some round sphere with center at O,*

(3) *For all $x\in S^{n-1}$, $t\in[0, 1]$, $\theta(x, F(x, t))<13n\epsilon/2+15\epsilon$.*

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