REPRESENTATION THEOREMS ON BANACH FUNCTION SPACES

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Let L_ρ be a Banach function space, i.e. a Banach space of (equivalence classes of) measurable point functions on a σ-finite measure space (Ω, Σ, μ), with ρ being a function norm possessing at least the weak Fatou property. The results obtained concern integral representations of bounded linear operators from a Banach space $\mathfrak{X}$ to L_ρ and from L_ρ (or a subspace) to $\mathfrak{X}$. These results in some cases complement and in other cases generalize work done in [1], [3], [5], [6], [7], [12], [13].

General notation and results on Banach function spaces can for the most part be found in the first parts of [11]; more detailed work is in [9]. A few further definitions are needed here. If $\mathfrak{X}$ and $\mathfrak{Y}$ are Banach spaces, let $B(\mathfrak{X}, \mathfrak{Y})$ be the space of bounded linear operators from $\mathfrak{X}$ to $\mathfrak{Y}$. Distinguish two subrings of Σ as $\Sigma_0 = \{E \in \Sigma : \rho(\chi_E) < \infty\}$ and $\Sigma_0' = \{E \in \Sigma : \rho'(\chi_E) < \infty\}$. A *partition* $\mathcal{E}$ is defined to be a finite disjoint collection of non-μ-null members of Σ_0 which are of finite measure. The *"averaged" step function* of a member f of L_ρ is defined as

$$f_{\mathcal{E}} = \sum_{\mathcal{E}} \left(\int_{E_i} |f| \, d\mu / \mu(E_i) \right) \chi_{E_i}.$$

A function norm ρ is said to have property (J) if, for each partition $\mathcal{E}$, $\rho(f_{\mathcal{E}}) \leqq \rho(f)$. (This is very similar to the *levelling property* of [5].)

1. The structure of the space $B(\mathfrak{X}, L_\rho)$.

DEFINITION 1. We define a space of set functions: $\mathcal{V}_\rho = \{x^*(\cdot) \mid x^*(\cdot) : \Sigma_0' \to \mathfrak{X}^*,\ x^*(\cdot)x$ is countably additive and μ-continuous for each $x \in \mathfrak{X}$, and $V_\rho(x^*(\cdot)) < \infty\}$ where

$$V_\rho(x^*(\cdot)) = \sup_{\|x\| \leqq 1} \sup_{\mathcal{E}} \rho\left(\sum_{\mathcal{E}} \frac{x^*(E_i)x}{\mu(E_i)} \chi_{E_i} \right).$$

The representation of bounded linear operators from $\mathfrak{X}$ to L_ρ is made in terms of this space.

[1] The results announced here are contained in the author's doctoral dissertation written at Carnegie Institute of Technology under the guidance of Professor M. M. Rao.