ANALYTIC DOMINATION BY FRACTIONAL POWERS OF A POSITIVE OPERATOR

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Introduction. Let A be an (unbounded) linear operator on a Banach space $\mathfrak{H}$. An *analytic vector* for A is an element $u \in \mathfrak{H}$ such that $A^n u$ is defined for all n and

$$\sum_{n=0}^{\infty} \frac{\|A^n u\|}{n!} t^n < \infty$$

for some $t > 0$, i.e. the power series expansion of $e^{tA}u$ is defined and has a positive radius of absolute convergence.

Nelson [2] introduced and studied the notion of *analytic domination* of one operator (or a family of operators) by another: A analytically dominates the operator X if every analytic vector for A is an analytic vector for X. In §1 we announce an analytic domination theorem; the hypotheses were suggested by Nelson's treatment of Lie algebras of skew-symmetric operators in [2], while the conclusion was suggested by some results of Kotake and Narasimhan [1]. We apply our theorem in §2 to the characterization of analytic vectors for a unitary representation of a Lie group.

1. **Analytic domination.** Let $\mathfrak{H}$ be a complex Hilbert space, and A a positive, selfadjoint operator on $\mathfrak{H}$, which we normalize so that $A \geqq I$. If α is a complex number, the operator A^α is defined via the operational calculus for selfadjoint operators, and $\mathfrak{D}(A^\alpha) \subseteq \mathfrak{D}(A^\beta)$ if $\operatorname{Re} \alpha \geqq \operatorname{Re} \beta$. (For any operator T on $\mathfrak{H}$, $\mathfrak{D}(T)$ will denote its domain of definition.) Let

$$\mathfrak{H}^\infty = \bigcap_{n=1}^{\infty} \mathfrak{D}(A^n)$$

(the C^∞-vectors for A). Then we have the following analytic domination criterion: $(\operatorname{ad} X(A) = XA - AX)$.

THEOREM 1. *Let $X: \mathfrak{H}^\infty \to \mathfrak{H}^\infty$ be symmetric or skew-symmetric. Suppose that for some α, $0 < \alpha < 1$,*

$$\|Xu\| \leqq \|A^\alpha u\|, \tag{1}$$

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