ON MEAN-PERIODICITY

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Introduction. Wiener's classical results on the closure of the translates of a function belonging to a Lebesgue class lead to the notion of mean-periodicity: A function f belonging to a given space M of functions defined on a Euclidean space is said to be *mean-periodic* in the space M if the closed linear span of the translates of f falls short of the entire space M. This notion clearly depends very strongly upon the function space M under consideration. The space M most thoroughly studied to date is the space of all continuous functions on the real line R under the topology of uniform convergence on compact sets [4], [5], [8].

It is the purpose of the present note to outline some new results concerning spectral analysis and synthesis in translation invariant subspaces connected with a certain space of functions holomorphic on R.

A certain function space and its dual. We consider at first the following space M. Let $f(x+iy)$ denote a function holomorphic in a closed strip $|y| \leqq 1/h$ for some $h < \infty$ such that

$$\sup_{|y| \leqq 1/h} \left\| e^{|x|/k} f(x+iy) \right\| < \infty \tag{1}$$

for every $k > 0$, where the norm appearing is that of $L^2(R)$. For h and k fixed, let S_k^h denote the Banach space formed by all such functions f with norm given by the expression (1). The space M is then the set

$$\bigcup_{h<\infty} \bigcap_{k>0} S_k^h$$

with the natural topology

$$M = \operatorname*{ind\,lim}_{h \to \infty} \ \operatorname*{proj\,lim}_{k \to 0^+} S_k^h.$$

The space M resembles spaces considered by Gelfand and Silov [2], [3] and by Roumieu [7], but it does not coincide with any of these. The space M does occur amongst the spaces investigated by Palamodov [6].

It turns out that M possesses several convenient properties as a topological vector space. M is a Montel space and hence reflexive. M