INVOLUTIONS WITH NONZERO ARF INVARIANT

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Browder and Livesay [1] have associated with each differentiable fixed point free involution $T: \Sigma^{2q+1} \to \Sigma^{2q+1}$, where $\Sigma = \Sigma^{2q+1}$ is a homotopy sphere, a "signature" $\sigma(\Sigma, T) \in Z(\equiv 0 \bmod 8)$ if q is odd, or an "Arf invariant" $c(\Sigma, T) \in Z_2$, if q is even. If $q \geqq 3$, then Σ contains a differentiably imbedded $2q$-sphere invariant with respect to T if and only if $\sigma(\Sigma, T) = 0$ or $c(\Sigma, T) = 0$ [1].

S. López de Medrano has constructed for every odd q examples of involutions T with nonzero signature. We prove the following

THEOREM. *For every* $k \geqq 1$ *there exists a fixed point free differentiable involution* $T: \Sigma^{4k+1} \to \Sigma^{4k+1}$ *with* $c(\Sigma, T) \neq 0$. *Here* Σ *is the "Kervaire homotopy sphere," i.e., the generator of* bP_{4k+2} [3], [4] *if the latter group is* $\neq 0$; *otherwise it is the standard sphere.*

The author understands that D. Montgomery and C. T. Yang have an example of a differentiable involution on Σ^9 with $c(\Sigma, T) \neq 0$. The fact that there are PL-involutions with $c(\Sigma, T) \neq 0$ on any $(4k+1)$-dimensional sphere follows from the classification of C. T. C. Wall [8].

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1. **Recall of definitions.** Let T be a differentiable (or PL) fixed point free involution on $\Sigma = \Sigma^{4k+1}$. A characteristic manifold N^{4k} is an invariant submanifold such that $\Sigma = A \cup B$, $N = A \cap B$, $B = TA$. There always exists such an N which is $(2k-1)$-connected [1]. Let $G = H_{2k}(N, Z_2) = H_{2k}(N) \otimes Z_2$. For $x, y \in G$, the intersection coefficients $A(x, y) = x \cdot y \in Z_2$, and $B(x, y) = x \cdot Ty$ define nonsingular symmetric bilinear forms on G and

$$(1) \qquad A(x, Ty) = B(x, y).$$

Browder and Livesay [1] define a quadratic form $\psi_0: G \to Z_2$ (if $x \in G$ is represented by an immersed sphere σ in general position with respect to $T\sigma$, then $\psi_0(x)$ is 1 if and only if $\sigma \cap T\sigma$ consists of an odd number of pairs of points). One can also define [7] another qua-

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