SEMICONTRACTIVE AND SEMIACCRETIVE NONLINEAR MAPPINGS IN BANACH SPACES

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Let X be a real Banach space, U and T mappings of a subset G of X into X. Then U is said to be nonexpansive if for all u and v in G,

$$\|U(u) - U(v)\| \leqq \|u - v\|, \tag{1}$$

while T is said to be accretive if for all u and v of G,

$$(T(u) - T(v), J(u - v)) \geqq 0, \tag{2}$$

where J is a mapping of X into its adjoint space X^* such that for all u in X, $(J(u), u) = \|u\|^2$ and $\|J(u)\| = \|u\|$.

In some recent papers ([7], [8], [9]), we have presented an existence theory for solutions of nonlinear functional equations in uniformly convex Banach spaces X involving nonexpansive and accretive mappings. These results were obtained by interweaving the fixed point theory of nonexpansive mappings with the theory of initial value problems for differential equations in X involving accretive operators. It is our object here to sharpen this theory and to use the sharpened form to extend the preceding results to more general classes of operators obtained by compact perturbation from nonexpansive or accretive operators. When X is a Hilbert space (or, more generally, has a weakly continuous duality mapping), such results were obtained earlier by the writer in [1], [2], [4]. The methods used there involving monotone operators do not apply in our more general context.

We begin by defining two basic classes of nonlinear mappings, the first generalizing the mappings of the form $U+C$ with U nonexpansive and C completely continuous, and the second, the mappings of the form $T+C$ with T accretive and C completely continuous. (We recall that a map C of X into X is said to be completely continuous if it carries weakly convergent sequences in X into strongly convergent sequences in X.)

DEFINITION 1. *Let X be a Banach space, G a subset of X, U a mapping of G into X. Then U is said to be semicontractive if there exists a mapping V of $G \times G$ into X such that $U(u) = V(u, u)$ for u in G, while:*

(a) *For each fixed v in G, $V(\cdot, v)$ is nonexpansive from G to X.*

(b) *For each fixed u in G, $V(u, \cdot)$ is completely continuous from G*