ON THE EQUATION $f^n+g^n=1$. II

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In [3], [4] the author proved that

$$f^3 + g^3 = 1 \tag{1}$$

has the solutions

$$f = \frac{1}{2}\left(1 + \frac{\wp'}{\sqrt{3}}\right)/\wp, \qquad g = \frac{1}{2}\left(1 - \frac{\wp'}{\sqrt{3}}\right)/\wp, \tag{2}$$

where the $\wp$-function satisfies

$$(\wp')^2 = 4\wp^3 - 1 \qquad (\text{i.e., } g_2 = 0 \quad \text{and} \quad g_3 = 1). \tag{3}$$

In [3] the author conjectured that all meromorphic solutions of (1) are necessarily elliptic functions of entire functions.

The conjecture was proved by Baker [1]. Baker proved

THEOREM 1. *Any functions $F(z)$ and $G(z)$, which are meromorphic in the plane and satisfy* (1) *have the form*

$$F = f(h(z)), \qquad G = \eta g(h(z)) = \eta f(-h(z)) = f(-\eta^2 h(z)),$$

where f and g are the elliptic functions in (2). *$h(z)$ is an entire function of z and η is a cube-root of unity.*

In this note we give an alternate proof of Theorem 1. In fact we prove

THEOREM 2. *The function f in* (2) *is a uniformizing function of the Riemann surface of*

$$x^3 + y^3 = 1. \tag{4}$$

f maps the whole z-plane in a 1-1 *manner on the Riemann surface of* (4).[1]

PROOF. Since $(1-f^3)^{1/3}$ is a single valued meromorphic function, one can easily verify that either f is a uniformizing function or $f=l(h)$, where l is a uniformizing function and h is nonlinear and entire. The latter, however, is impossible. For otherwise we have $f=l(h)$ and $g=r(h)$, so that f and g and hence $\wp$ and $\wp'$ have h as a common

[1] In the sequel, uniformizing functions are assumed to have this additional property.