THE TOPOLOGICAL DEGREE AND GALERKIN APPROXIMATIONS FOR NONCOMPACT OPERATORS IN BANACH SPACES

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Let X and Y be real Banach spaces, G a bounded open subset of X, $\mathrm{cl}(G)$ its closure in X, $\mathrm{bdry}(G)$ its boundary in X. We consider mappings T, (nonlinear, in general), of $\mathrm{cl}(G)$ into Y which are *A-proper*, in the sense defined below, with respect to a given approximation scheme of generalized Galerkin type. We define a generalized concept of *topological degree* for such mappings with respect to the given approximation scheme, and show that this degree (which may be multivalued) has the basic properties of the classical Leray-Schauder degree (where the latter is defined on the narrower class of maps of X into X of the form $I+C$, with I the identity and C compact).

For a wide class of A-proper mappings T of the form $T=H+C$, with H an A-proper homeomorphism of a suitable type and C compact, we show that the degree is single-valued and coincides with another generalized degree studied in Browder [9] and Browder-Nussbaum [11], and in particular is independent of the approximation scheme involved. In particular, this holds if H is *strongly accretive* from X to X (cf. Browder [4], [5], [6], [8]), including as a very special case all maps H of the form $H=I-U$, with U a strict contraction.

DEFINITION 1. *Let X and Y be Banach spaces. By an (oriented) approximation scheme for mappings from X to Y, we mean: an increasing sequence $\{X_n\}$ of oriented finite dimensional subspaces of X, an increasing sequence $\{Y_n\}$ of oriented finite dimensional subspaces of Y, and a sequence of linear projection maps $\{Q_n\}$ with Q_n mapping Y on Y_n such that* $\dim(X_n)=\dim(Y_n)$ *for all n, $\bigcup_n X_n$ is dense in X, and $Q_n y\to y$ as $n\to\infty$ for all y in Y.*

DEFINITION 2. *Let G be a bounded open subset of X, T a mapping of* $\mathrm{cl}(G)$ *into Y. Then T is said to be A-proper with respect to a given approximation scheme in the sense of Definition* 1 *if for any sequence $\{n_j\}$ of positive integers with $n_j\to\infty$ and a corresponding sequence $\{x_{n_j}\}$ in* $\mathrm{cl}(G)$ *with each x_{n_j} in X_{n_j} such that $Q_{n_j}Tx_{n_j}$ converges strongly in Y to an element y, there exists an infinite subsequence $\{n_{j(k)}\}$ such that $x_{n_{j(k)}}$ converges strongly to x in X as $k\to\infty$ and $T(x)=y$.*

The concept of A-proper mapping is a slight variant of the condition (H) of Petryshyn [18], and both are modifications of the definition of *P-compact* mapping in Petryshyn [15], [16], and [17]. A sim-