DIFFRACTION BY A HYPERBOLIC CYLINDER

BY CLIFFORD O. BLOOM[1]

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We investigate the asymptotic behavior as $K\to\infty$ of the solution $G(x, y; x_0, y_0: K)$ of the following scattering problem P:

(i) $[\Delta+K^2]U=\delta(x-x_0, y-y_0)$, $(x, y), (x_0, y_0)\in D$;

(ii) $\partial_n U=0$, $(x, y)\in C$;

(iii) $\lim_{\rho\to\infty}\int_{\Sigma(\rho,0)\cap\bar{D}} |\partial U/\partial r-iKU|^2dS=0$.

Here C is the left branch of the coordinate hyperbola $(x/h\cdot\cos\mathbf{n})^2-(y/h\cdot\sin\mathbf{n})^2=1$, $\pi/2<\mathbf{n}<\pi$. In parametric form C is given by the equations $x=h\cdot\cos\mathbf{n}\cdot\cosh\xi$, $y=\pm h\cdot\sin\mathbf{n}\cdot\sinh\xi$, $\xi\geqq 0$.

D is the infinite two dimensional region bounded by the convex side of C; D consists of all points (x, y) with elliptic coordinates (ξ, η) such that $\xi\geqq 0$, and $-\mathbf{n}\leqq\eta\leqq\mathbf{n}$. $\bar{D}=D\cup C$, and $\Sigma(\rho, 0)=\{(x,y):x^2+y^2=\rho^2\}$.

Δ is the two dimensional Laplacian, $\delta(x-x_0, y-y_0)$ is Dirac's δ-function, and ∂_n denotes differentiation in the direction of the outward normal to C.

Our result is a rigorous asymptotic expansion of the Green's function G as $K(>0)\to\infty$ that holds uniformly in every closed bounded subset $S_<(x_0, y_0)$ of the "shadow" $S(x_0, y_0)$ of C. $S(x_0, y_0)$ consists of those points in $D\cup C$ that cannot be joined to (x_0, y_0) by a line segment lying entirely in $D\cup C$.

The asymptotic expansion we get for G confirms the geometrical theory of diffraction by convex cylinders of infinite cross section (see [1]).

Furthermore, our rigorous asymptotic solution of the problem P can be used with certain bounds to obtain asymptotic solutions of a general class of scattering problems with smooth convex boundaries C' that coincide with C in neighborhoods of the points of "diffraction"; the points where the boundary of $S(x_0, y_0)$ intersects C. For example if C' is formed by joining a convex arc A to the "illuminated" part of C, then an asymptotic expansion of the solution G' in the shadow $S'(x_0, y_0)$ $(=S(x_0, y_0))$ can be obtained once it is known, for some positive N, that $G'(x, y; x_0, y_0: K)=O(K^N)$ as $K\to\infty$, uniformly in (x, y), $(x, y)\in A$.

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