

## RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited. Manuscripts more than eight typewritten double spaced pages long will not be considered as acceptable.

### WALL'S SURGERY OBSTRUCTION GROUPS FOR $Z \times G$ , FOR SUITABLE GROUPS $G$

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**Introduction.** Let  $\mathcal{C}$  be the category whose objects are pairs  $(G, w)$ ,  $G$  a group and  $w$  a homomorphism of  $G$  into  $Z_2$ , and whose morphisms are the obvious ones. Every finite Poincaré complex  $X^n$  determines functorially an element  $(\pi_1(X), w(X))$  of this category; let  $w(X)(b) = 1$  if  $b$  preserves orientation and let  $w(X)(b) = -1$  otherwise. There is a sequence of functors  $L_n$ ,  $n \geq 5$ , from  $\mathcal{C}$  to the category of abelian groups, with  $L_n = L_{n+4}$  all  $n \geq 5$ , which plays the role of the range of a surgery obstruction. More precisely, let  $X^n$  be a compact smooth manifold and let  $v$  be its stable normal bundle. (Actually, one only needs a finite Poincaré complex with a given vector bundle; see [3] and [4].) Let  $\Omega_n(X, v)$  be the cobordism classes of triples  $(M, \phi, F)$ ,  $M$  a compact smooth manifold,  $\phi: (M, \partial M) \rightarrow (X, \partial X)$  a map of degree one which induces a homotopy equivalence of boundaries, and  $F$  a stable framing of  $\iota(M) \oplus \phi^*v$ , where  $\iota M =$  tangent bundle of  $M$ . Then for  $n \geq 5$ , there is a map  $\theta: \Omega_n(X, v) \rightarrow L_n(\pi_1(X), w(X))$  such that  $\theta[M, \phi, F] = 0$  if and only if this class  $[M, \phi, F]$  contains  $(N, \psi, G)$  with  $\psi$  a homotopy equivalence. For  $n$  even, the functors  $L_n$  and this map are defined by Wall in [3]. For  $n$  odd, they are defined by Wall in [4], but with "homotopy equivalence" replaced by "simple homotopy equivalence." However, one can slightly alter the procedures of [4] to define  $L_n$  and  $\theta$  with the properties just mentioned.

The groups  $L_n(\pi_1 X, wX)$  are not too large in the sense that every one of their elements is the obstruction to some surgery problem with boundary. In fact, we have the following result, due essentially to Wall (see [3, p. 274] and [4, §5, 6]).

**THEOREM 0.1.** *Let  $X^{m-1}$ ,  $m \geq 6$ , be a compact connected smooth manifold. Let  $v$  be the stable normal bundle of  $X$ . Let  $\gamma$  be a given element of  $L_m(\pi_1 X, wX) = L_m(\pi_1(X \times I), w(X \times I))$ . Let  $\phi_1$  be a homotopy equivalence of  $M^{m-1}$  and  $X$ ,  $M$  a compact smooth manifold, which induces a homotopy equivalence of boundaries. Let  $F_1$  be a stable framing of*